

Visit
FREETESTPAPER.com
for more papers



Website: freetestpaper.com



[Facebook.com/freetestpaper](https://www.facebook.com/freetestpaper)



[Twitter.com/freetestpaper](https://www.twitter.com/freetestpaper)

Name: _____ ()

Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS

4052/01

Paper 1

Thursday 17 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

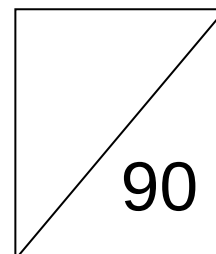
Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.



This document consists of **20** printed pages.



圣尼各拉女校

CHIJ ST. NICHOLAS GIRLS' SCHOOL

Girls of Grace • Women of Strength • Leaders with Heart

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 Given that $\frac{4}{128} = 2^a$, find the value of a .

Answer $a = \dots\dots\dots$ [1]

- 2 Ms Li takes a loan of \$12000 from a bank that charges a simple interest rate of 3.5% per annum for 4 years.
Calculate the total amount she has to pay the bank at the end of 4 years.

Answer \$..... [2]

- 3 Store A sells a spice blender for \$124.90 with a 10% discount.
Store B sells the same blender for \$115.00 with a 12% discount.

- (a) By rounding the prices to 2 significant figures and the discounts to 1 significant figure, show that the discounted price in Store A and the discounted price in Store B are equal.

Show the numbers you use.

Answer

[1]

- (b) Without doing any further calculation, explain why the actual discounted price in Store B is lower than the estimated discounted price.

Answer

.....

..... [1]

- 4 The times, in minutes, taken by some students to complete an assignment are shown below.

49 53 45 44 53 49 59 y 60 50

- (a) If the modal time is 53 minutes, state the value of y .

Answer $y = \dots\dots\dots$ [1]

- (b) Hence, find the median time.

Answer $\dots\dots\dots$ minutes [1]

- 5 Simplify $\left(\frac{27x^9}{y^{12}}\right)^{\frac{2}{3}} \div \frac{x^5}{y^7}$.

Answer $\dots\dots\dots$ [3]

- 6 Write as a single fraction in its simplest form $\frac{12-4x}{(x-3)^2} - 1$.

Answer $\dots\dots\dots$ [3]

- 7 In a school club, 72 of the members are boys and 48 are girls.

Calculate the least number of additional girls that would need to join the club in order to raise the percentage of girls to greater than 45%.

Answer [2]

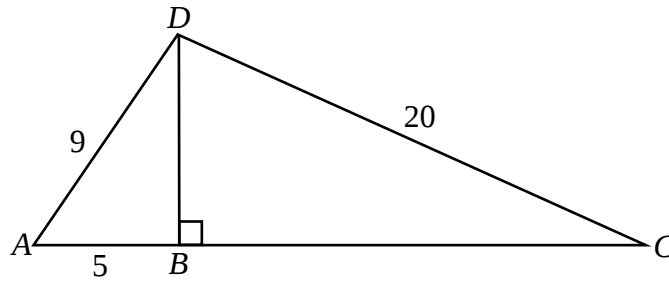
- 8 (a) Solve the inequalities $-8.5 \leq 3x - 4 < 11$.

Answer [2]

- (b) Write down all the integers that satisfy $-8.5 \leq 3x - 4 < 11$.

Answer [1]

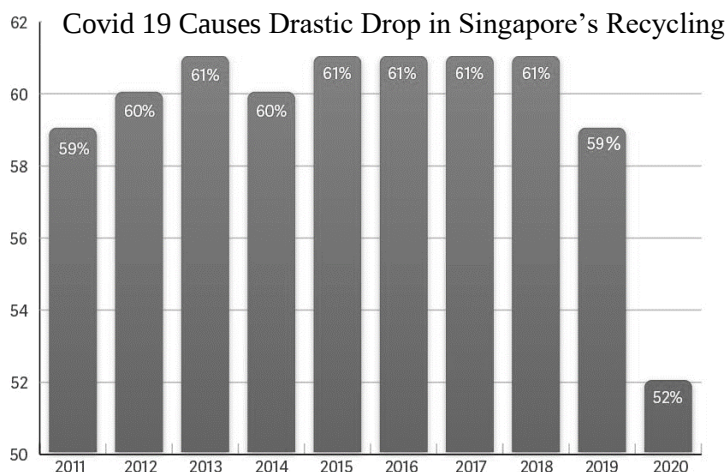
9



In the triangle ACD , $AB = 5$ cm, $AD = 9$ cm, $CD = 20$ cm and BD is perpendicular to AC .
Calculate BC .

Answer $BC = \dots\dots\dots$ cm [2]

10



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

.....

..... [2]

- 11 (a)** Expand and simplify $(5x-14)(5x+14)$.

Answer [2]

- (b)** Given that x is a positive integer and $(5x-14)(5x+14)$ is a prime number between 20 and 30, find this prime number.

Answer [1]

- 12** John, Eric and Kate walk from point A to point B along the same route.

Eric takes 10 minutes 30 seconds to complete this journey.

John takes 10% less than the time taken by Eric.

Kate takes 20% more than the time taken by Eric.

- (a)** Calculate the time taken by Kate in seconds.

Answer seconds [1]

- (b)** Express the ratio of the times taken by John, Eric and Kate in the form $a:b:c$, where a , b and c are integers in their lowest terms.

Answer : : [2]

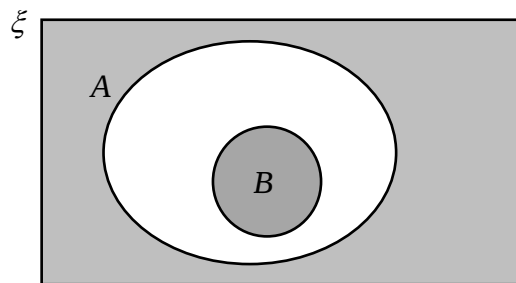
- 13 (a) $\xi = \{\text{integers } x : 2 \leq x \leq 18\}$
 $M = \{\text{prime numbers}\}$
 $N = \{\text{factors of 28}\}$

Draw a Venn diagram to illustrate this information.

Answer

[2]

- (b) Use set notation to describe the shaded region.



Answer [1]

- (c) The sets ξ , P and Q satisfy the conditions $n(\xi) = 18$, $n(P) = 9$ and $n(Q) = 15$.
Find the largest possible value of $n(P \cup Q)'$.

Answer [1]

- 14 (a)** Written as a product of its prime factors, $504 = 2^3 \times 3^2 \times 7$.
 p and q are both prime numbers where $p < q$ and the number $\frac{504p}{q}$ is a perfect square.

Find the value of p and the value of q

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [2]

- (b)** A rectangle has an area of 200 cm^2 . The length is $x \text{ cm}$ and the breadth is $y \text{ cm}$.
 Given that x and y are integers and $x > y \geq 10$, find the value of x and the value of y .

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [2]

- 15 (a) Factorise completely $7xy - 10 - 5x + 14y$.

Answer [2]

- (b) Using factorization, solve $63x^2 - 4x - 4 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- 16 Two geometrically similar models of a house are made using the same material.
The mass of the smaller model is 750 g while the mass of the larger model is 6 kg.

- (a) Find the ratio of the height of the smaller model to the height of the larger model in the form $1 : n$.

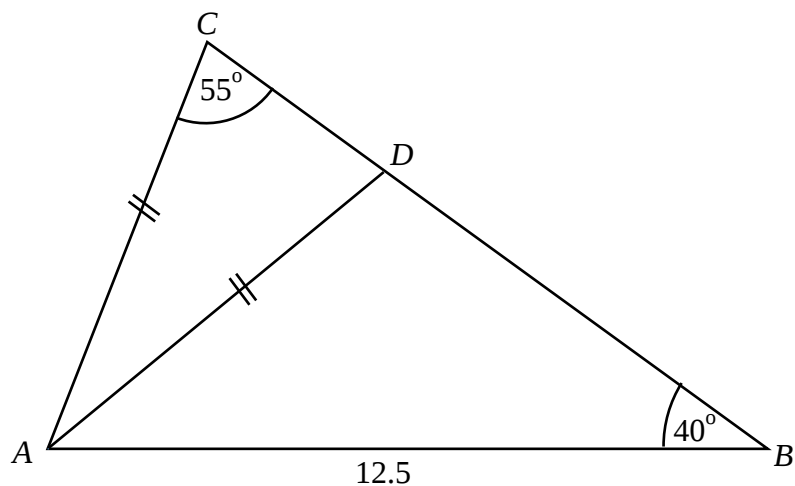
Answer [2]

- (b) The larger model is made to a scale of $3 : 4000$.
The length of the actual house is 120 m.

Find the length of the house in the smaller model.

Answer cm [2]

17



In the triangle ABC , $AB = 12.5$ cm, angle $ABC = 40^\circ$, angle $ACB = 55^\circ$ and $AC = AD$.

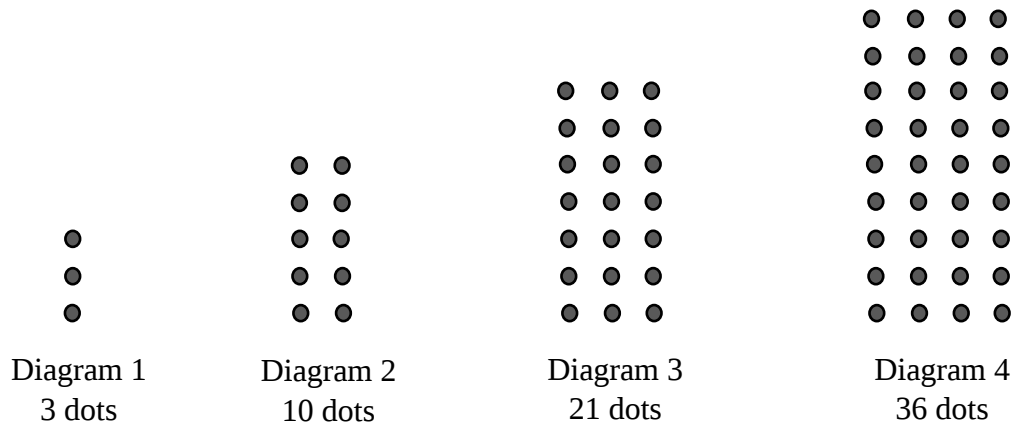
(a) Calculate AC .

Answer $AC = \dots\dots\dots$ cm [2]

(b) Calculate the area of triangle ADB .

Answer $\dots\dots\dots$ cm^2 [3]

- 18 The first four diagrams in a sequence are shown below.



- (a) Find the number of dots in Diagram 5.

Answer [1]

- (b) Find an expression, in terms of n , for the number of dots in Diagram n .

Answer [2]

- (c) Find the difference in the number of dots between Diagram 7 and Diagram 6.

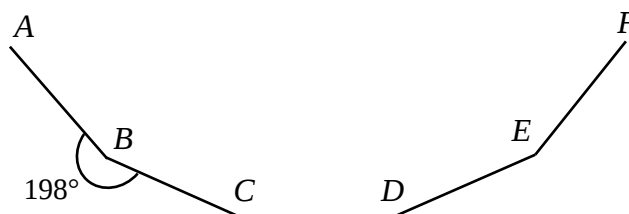
Answer [1]

- (d) Show that the difference in the number of dots between Diagram $(n + 1)$ and Diagram n is $4n + 3$.

Answer

[2]

19



The diagram shows part of a regular n -sided polygon, $ABCDEF\dots$
 Given that the reflex angle $ABC = 198^\circ$

- (a) Prove that triangles ABC and triangle CDE are congruent.

Answer

[2]

- (b) Find the size of each exterior angle.

Answer [1]

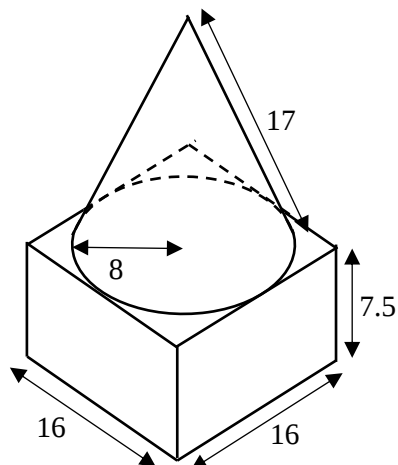
- (c) Find the value of n .

Answer $n = \dots\dots\dots$ [1]

- (d) Find the angle BEF .

Answer Angle $BEF = \dots\dots\dots$ [2]

20



The diagram shows a solid formed from a cone and a cuboid.

The cone has a radius of 8 cm and slant height of 17 cm.

The cuboid has a square base of length 16 cm and a height of 7.5 cm.

(a) Calculate the volume of the solid figure.

Answer cm^3 [3]

(b) Calculate the total surface area of the solid figure.

Answer cm^2 [3]

- 21** At bakery A, a chocolate bread loaf costs \$2.50, a wholemeal bread loaf costs \$3 and a raisin bread loaf costs \$3.80 .

At bakery B, a chocolate bread loaf costs \$3.50, a wholemeal bread loaf costs \$3.20 and a raisin bread loaf costs \$3.

This information can be represented by the matrix $\mathbf{P} = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} C \\ W \\ R \end{matrix} & \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix} \end{matrix}$

- (a)** Janet buys 2 loaves of chocolate bread, 4 loaves of wholemeal bread and 3 loaves of raisin bread.
 Karen buys 5 loaves of chocolate bread, 3 loaves of wholemeal bread and x loaves of raisin bread.
 Represent this information in a 2×3 matrix, $\mathbf{N}$.

Answer $\mathbf{N} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

- (b)** Find, in terms of x , the matrix $\mathbf{S} = \mathbf{NP}$.

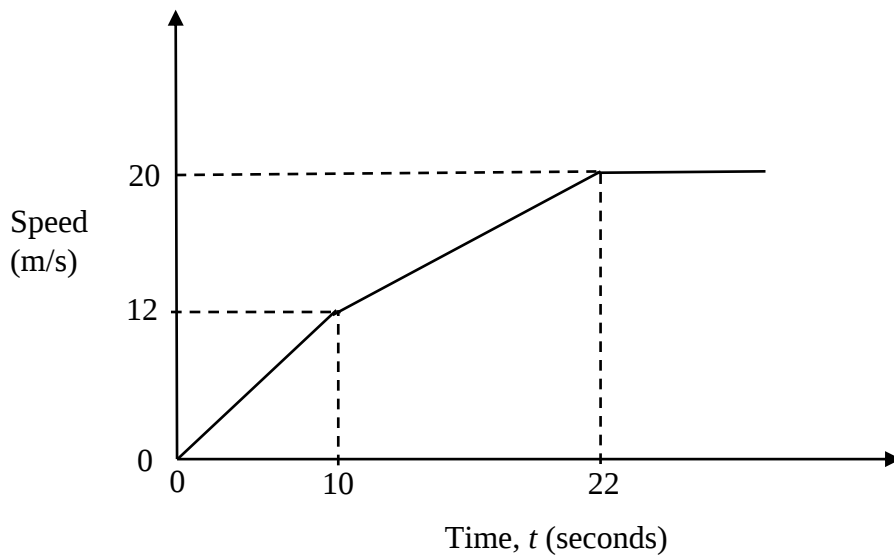
Answer $\mathbf{S} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [2]

- (c)** Karen has a budget of \$30.
 By writing an inequality in x , determine the total number of loaves of bread she can buy from bakery B.

Answer loaves [2]

- 22 The diagram shows the speed-time graph of a car journey.

The car accelerates for 10 seconds reaching a speed of 12 m/s. It then accelerates for another 12 seconds and then travels at a constant speed of 20 m/s.



- (a) Calculate the speed of the car when $t = 13$.

Answer m/s [2]

- (b) Calculate the distance travelled in the first 22 seconds.

Answer m [2]

- (c) A motor cyclist starts 13 seconds later from the same place as the car, and accelerates uniformly for 9 seconds until it reaches a speed of 25 m/s.

Is the motor cyclist behind the car at $t = 22$?
Justify your decision.

Answer

[2]

- 23 The table shows the time taken by 100 females to complete the 2.4 km run.

Time (t minutes)	Frequency
$8 \leq t < 10$	6
$10 \leq t < 12$	25
$12 \leq t < 14$	36
$14 \leq t < 16$	29
$16 \leq t < 18$	4

- (a) Calculate an estimate for
- (i) the mean time for the females,

Answer minutes [1]

- (ii) the standard deviation of their timing.

Answer minutes [1]

- (iii) The mean time for another group of 100 males was 12.6 minutes and the standard deviation was 2.5 minutes.

Make two comparisons between the times for males and the times for females.

Answer

.....

.....

.....

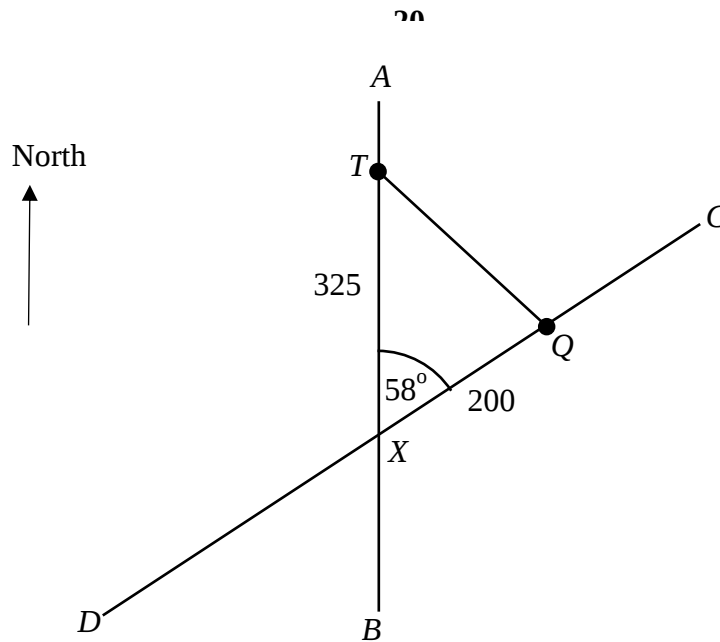
.....

..... [2]

- (b) Two female runners are chosen at random.

Find the probability that both runners complete the 2.4 km run in less than 12 minutes.

Answer [2]



AB and CD are two roads which meet at point X at an angle of 58° , with A due north of X . Point Q is 200 km from point X .

At 1040, a truck T is 325 km north of point X and is travelling at 75 km/h along the path TQ to reach point Q .

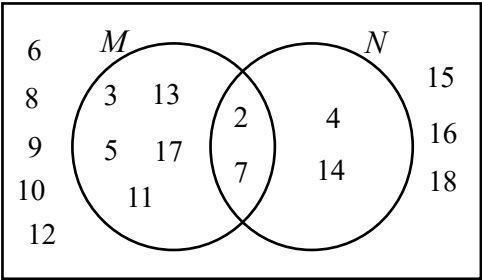
- (a) At what time would the truck reach point Q ?

Answer [3]

- (b) Calculate the bearing of Q from T .

Answer [2]

Answer 2023 MA Prelim Paper 1

1. $a = -5$
2. \$13680
3. (a) Show both \$108
(b) Actual price (\$115) is lower than estimated price (\$120), actual discount (12%) is higher than estimated discount (10%).
This explains why the actual discounted price is lower than the estimated discounted price.
4. (a) 53
(b) 51.5
5. $\frac{9x}{y}$
6. $\frac{x+1}{3-x}$
7. 11
8. (a) $-1.5 \leq x \leq 5$
(b) -1, 0, 1, 2, 3, 4
9. 18.5
10. The title is biased or the title uses loaded words like drastic drop. This leads to misinterpretation because it does not allow the reader to make own judgement.
Or The vertical axis starts from 50 instead of 0. This leads to misinterpretation because it exaggerates the differences between the years.
11. (a) $25x^2 - 196$
(b) 29
12. (a) 756
(b) 9:10:12
13. (a) ξ

(b) $B \cup A'$
(c) 3
14. (a) $p = 2, q = 7$
(b) $x = 20, y = 10$
15. (a) $(7y - 5)(x + 2)$
(b) $x = \frac{2}{7}, -\frac{2}{9}$
16. (a) 1:2
(b) 4.5 cm

17. (a) 9.81 cm

(b) 15.9 cm²

18. (a) 55

(b) $n(2n+1)$

(c) 27

(d) show $(n+1)(2(n+1)+1)-n(2n+1)$ leading to $4n+3$

19. (a) show with reasons (mention regular polygon)

(b) 18°

© 20

(d) 144o

20. (a) 2930 cm³

(b) 1220 cm²

21. (a) $N = \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix}$

(b) $= \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix} \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix}$

$= \begin{pmatrix} 28.4 & 28.8 \\ 21.5+3.8x & 27.1+3x \end{pmatrix}$

(c) 8

22. (a) 14 m/s

(b) 252 m

(c) show 112.5 m is less than 252 m to justify that motor cyclist is behind the car

23. (a) (i) 13 min

(ii) 1.94 min

(iii) The males run faster as the mean time is lower than the mean time for females.

The timings for the females shows more consistency or less spread (not less range) or less variation as its standard deviation is lower than the standard deviation for the males.

(b) $\frac{31}{330}$

24. (a) 1422 or 1421

(b) 142.2°

Name: _____ ()

Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS **Student copy**

4052/01

Paper 1

Thursday 17 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

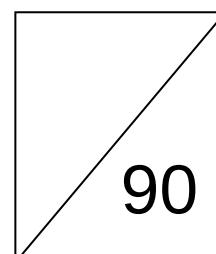
Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.



This document consists of **20** printed pages.



圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL

Girls of Grace • Women of Strength • Leaders with Heart

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 Given that $\frac{4}{128} = 2^a$, find the value of a .

$$\frac{4}{128} = \frac{1}{32} = 2^{-5}$$

$$\therefore a = -5$$

Answer $a = \dots\dots\dots$ [1]

- 2 Ms Li takes a loan of \$12000 from a bank that charges a simple interest rate of 3.5% per annum for 4 years.
Calculate the total amount she has to pay the bank at the end of 4 years.

$$\text{Simple interest} = \frac{12000 \times 3.5 \times 4}{100}$$

$$= \$1680$$

$$\text{Total amount} = 12000 + 1680$$

$$= \$13680$$

Answer \$..... [2]

- 3 Store A sells a spice blender for \$124.90 with a 10% discount.
Store B sells the same blender for \$115.00 with a 12% discount.

- (a) By rounding the prices to 2 significant figures and the discounts to 1 significant figure, show that the discounted price in Store A and the discounted price in Store B are equal.

Show the numbers you use.

Answer

$$\text{Store A } \$120 \times 0.9 = \$108$$

$$\text{Store B } \$120 \times 0.9 = \$108$$

[1]

- (b) Without doing any further calculation, explain why the actual discounted price in Store B is lower than the estimated discounted price.

Answer

The actual price (\$115) is lower than the estimated price (\$120). The actual discount percent (12%) is more than the estimated discount percent (10%). This gives a lower actual discounted price than the estimated discounted price.

[1]

- 4 The times, in minutes, taken by some students to complete an assignment are shown below.

49 53 45 44 53 49 59 y 60 50

- (a) If the modal time is 53 minutes, state the value of y .

44 45 49 49 50 53 53 y 59 60

Mode = 53

$y = 53$

Answer $y = \dots\dots\dots$ [1]

- (b) Hence, find the median time.

44 45 49 49 50 53 53 53 59 60

Middle position Median = $\frac{50+53}{2}$

at $\frac{10+1}{2} = 5.5th$ = 51.5

Answer $\dots\dots\dots$ minutes [1]

- 5 Simplify $\left(\frac{27x^9}{y^{12}}\right)^{\frac{2}{3}} \div \frac{x^5}{y^7}$.

$$= \left(\frac{3^3 x^9}{y^{12}}\right)^{\frac{2}{3}} \div \frac{x^5}{y^7}$$

$$= \frac{9x^6}{y^8} \times \frac{y^7}{x^5}$$

$$= \frac{9x}{y}$$

$\dots\dots\dots$ [3]

- 6 Write as a single fraction in its simplest form $\frac{12-4x}{(x-3)^2} - 1$.

$$\begin{aligned} &= \frac{4(3-x)}{(x-3)^2} - 1 \\ &= \frac{-4}{(x-3)} - 1 \\ &= \frac{-4-(x-3)}{(x-3)} \\ &= \frac{-x-1}{x-3} = \frac{x+1}{3-x} \end{aligned}$$

$$\begin{aligned} &= \frac{4(3-x)-(x-3)^2}{(x-3)^2} \\ &= \frac{12-4x-x^2+6x-9}{(x-3)^2} \\ &= \frac{-x^2+2x+3}{(x-3)^2} \\ &= \frac{(3-x)(1+x)}{(x-3)^2} \\ &= \frac{-(x-3)(1+x)}{(x-3)^2} \\ &= \frac{-(1+x)}{(x-3)} \end{aligned}$$

Answer

- 7 In a school club, 72 of the members are boys and 48 are girls.

Calculate the least number of additional girls that would need to join the club in order to raise the percentage of girls to greater than 45%.

Let x be the number of additional girls needed to join the club.

$$\frac{48+x}{48+72+x} > \frac{45}{100} \quad \text{or} \quad \frac{48+x}{48+72+x} > \frac{9}{20}$$

$$9(120+x) < 960 + 20x$$

$$1080 + 9x < 960 + 20x$$

$$11x > 120$$

$$x > 10.91, \text{ least number is } 11$$

Answer [2]

- 8 (a) Solve the inequalities $-8.5 \leq 3x - 4 < 11$.

$$\begin{array}{ll} -8.5 \leq 3x - 4 & 3x - 4 < 11 \\ 3x \geq -4.5 & 3x < 15 \\ x \geq -1.5 & x < 5 \\ \therefore -1.5 \leq x < 5 \end{array}$$

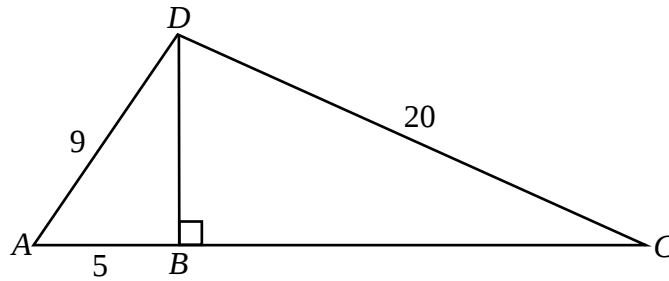
Answer [2]

- (b) Write down all the integers that satisfy $-8.5 \leq 3x - 4 < 11$.

$$-1, 0, 1, 2, 3, 4$$

Answer [1]

9

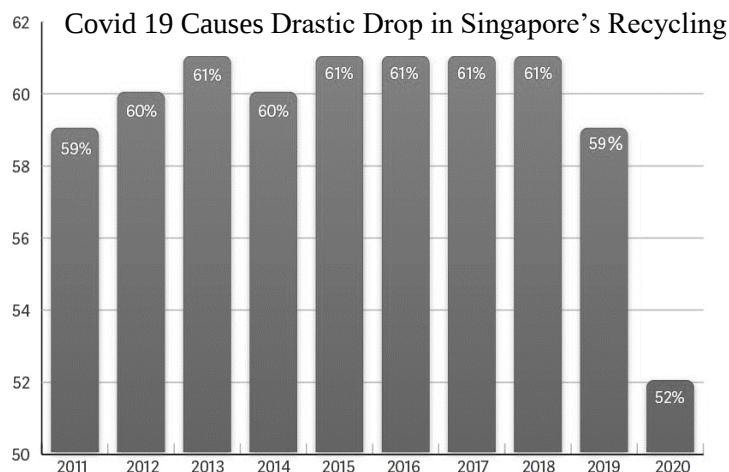


In the triangle ACD , $AB = 5$ cm, $AD = 9$ cm, $CD = 20$ cm and BD is perpendicular to AC . Calculate BC .

$$\begin{aligned}
 BD^2 &= 9^2 - 5^2 \\
 BD^2 &= 56 \\
 BD^2 + BC^2 &= 20^2 \\
 \therefore BC &= \sqrt{20^2 - 56} \\
 \therefore BC &= \sqrt{344} \\
 \therefore BC &= 18.5 \text{ cm (3sf)}
 \end{aligned}$$

Answer $BC = \dots\dots\dots$ cm [2]

10



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Vertical axis starts from 50 instead of 0.

It leads to misinterpretation because it exaggerates the differences between the years.

The title is biased. (Or the title uses loaded words “drastic drop”).

It leads to misinterpretation because it does not allow the reader to make own judgement.

- 11 (a) Expand and simplify $(5x-14)(5x+14)$.

$$\begin{aligned}(5x-14)(5x+14) \\ &= (5x)^2 - 14^2 \\ &= 25x^2 - 196\end{aligned}$$

Answer [2]

- (b) Given that x is a positive integer and $(5x-14)(5x+14)$ is a prime number between 20 and 30, find this prime number.

$$\begin{aligned}5x-14 &= 1 \\ 5x &= 15 \\ x &= 3 \\ \text{Prime number } 1 \times (5 \times 3 + 14) &= 29\end{aligned}$$

Answer [1]

- 12 John, Eric and Kate walk from point A to point B along the same route.

Eric takes 10 minutes 30 seconds to complete this journey.

John takes 10% less than the time taken by Eric.

Kate takes 20% more than the time taken by Eric.

- (a) Calculate the time taken by Kate in seconds.

$$\begin{aligned}\text{Kate's time} &= 1.2 \times 630 \text{ seconds} \\ &= 756 \text{ seconds}\end{aligned}$$

Answer seconds [1]

- (b) Express the ratio of the times taken by John, Eric and Kate in the form $a:b:c$, where a , b and c are integers in their lowest terms.

$$\begin{aligned}\text{John's time} &= 0.9 \times 630 \text{ seconds} \\ &= 567 \text{ seconds} \\ 567 : 630 : 756 \\ &= 9 : 10 : 12\end{aligned}$$

or

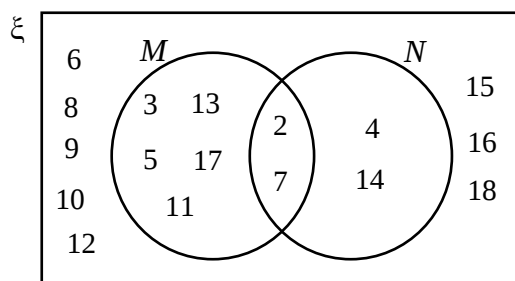
$$\begin{aligned}\text{John, Eric and Kate} \\ &= 90\% : 100\% : 120\% \\ &= 9 : 10 : 12\end{aligned}$$

Answer : : [2]

- 13 (a) $\xi = \{\text{integers } x : 2 \leq x \leq 18\}$
 $M = \{\text{prime numbers}\}$
 $N = \{\text{factors of 28}\}$

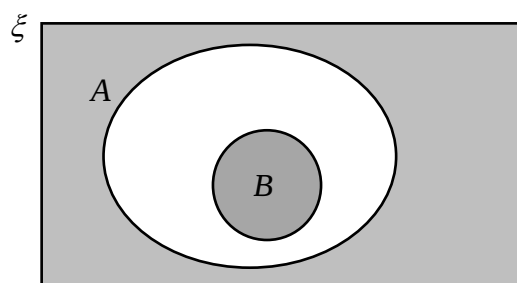
Draw a Venn diagram to illustrate this information.

Answer



[2]

- (b) Use set notation to describe the shaded region.



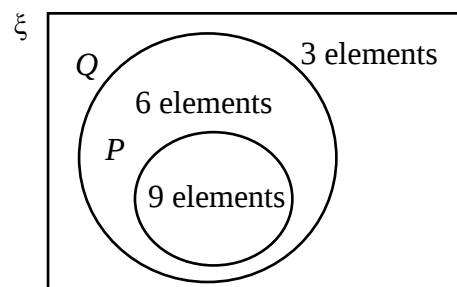
$B \cup A'$

Answer [1]

- (c) The sets ξ , P and Q satisfy the conditions $n(\xi) = 18$, $n(P) = 9$ and $n(Q) = 15$.

Find the largest possible value of $n(P \cup Q)'$.

Largest possible value of $n(P \cup Q)' = 3$



Answer [1]

- 14 (a) Written as a product of its prime factors, $504 = 2^3 \times 3^2 \times 7$.

p and q are both prime numbers where $p < q$ and the number $\frac{504p}{q}$ is a perfect square.

Find the value of p and the value of q

$$\frac{504p}{q} = 2^3 \times 3^2 \times 7 \times \frac{2}{7}, \quad p = 2, \quad q = 7$$

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [2]

- (b) A rectangle has an area of 200 cm^2 . The length is $x \text{ cm}$ and the breadth is $y \text{ cm}$.

Given that x and y are integers and $x > y \geq 10$, find the value of x and the value of y .

$$200 = 2^3 \times 5^2$$

$$x = 2^2 \times 5 = 20$$

$$y = 2 \times 5 = 10$$

$$200 \times 1$$

$$100 \times 2$$

$$50 \times 4$$

$$40 \times 5$$

$$25 \times 8$$

$$20 \times 10$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [2]

- 15 (a) Factorise completely $7xy - 10 - 5x + 14y$.

$$= 7xy + 14y - 5x - 10$$

$$= 7y(x + 2) - 5(x + 2)$$

$$= (x + 2)(7y - 5)$$

$$= 7xy - 5x - 10 + 14y$$

$$= x(7y - 5) - 2(5 - 7y)$$

$$x(7y - 5) + 2(7y - 5)$$

$$= (7y - 5)(x + 2)$$

Answer [2]

- (b) Using factorization, solve $63x^2 - 4x - 4 = 0$.

$$63x^2 - 4x - 4 = 0$$

$$(7x - 2)(9x + 2) = 0$$

$$x = \frac{2}{7} \quad \text{or} \quad x = -\frac{2}{9}$$

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- 16 Two geometrically similar models of a house are made using the same material.
The mass of the smaller model is 750 g while the mass of the larger model is 6 kg.

- (a) Find the ratio of the height of the smaller model to the height of the larger model in the form $1 : n$.

$$\frac{h_{\text{smaller}}}{h_{\text{larger}}} = \sqrt[3]{\frac{750}{6000}} \quad \text{or} \quad \left(\frac{h_{\text{smaller}}}{h_{\text{larger}}}\right)^3 = \frac{750}{6000}$$

$$= \frac{1}{2}$$

$$h_{\text{smaller}} : h_{\text{larger}} = 1 : 2$$

$$\frac{m_{\text{smaller}}}{m_{\text{larger}}} = \frac{\text{density} \times V_{\text{smaller}}}{\text{density} \times V_{\text{larger}}}$$

$$= \left(\frac{h_{\text{smaller}}}{h_{\text{larger}}}\right)^3$$

Answer [2]

- (b) The larger model is made to a scale of $3 : 4000$.
The length of the actual house is 120 m.

Find the length of the house in the smaller model.

$$\frac{l_{\text{larger}}}{12000} = \frac{3}{4000}$$

$$l_{\text{larger}} = \frac{3}{4000} \times 12000$$

$$= 9 \text{ cm}$$

$$\frac{l_{\text{larger}}}{l_{\text{smaller}}} = \frac{h_{\text{larger}}}{h_{\text{smaller}}} = \frac{2}{1}$$

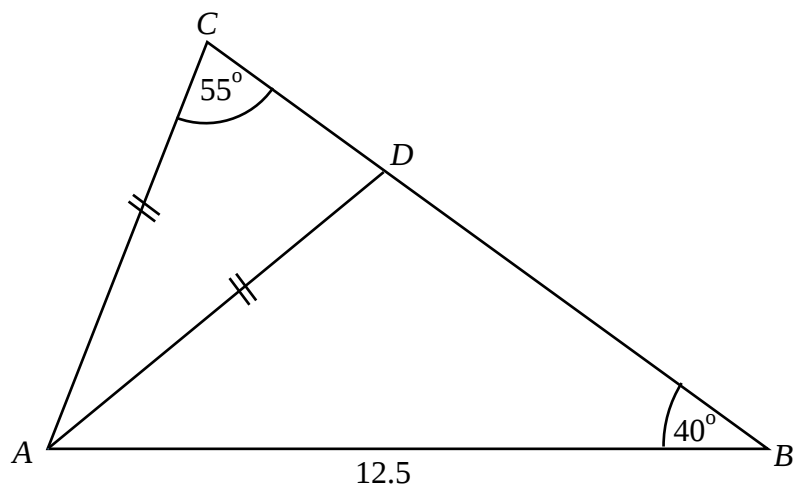
$$\frac{9}{l_{\text{smaller}}} = \frac{2}{1}$$

$$\therefore l_{\text{smaller}} = 4.5 \text{ cm}$$

Actual length 120m,
Large model length 9cm
Small model length 4.5 cm

Answer

17



In the triangle ABC , $AB = 12.5$ cm, angle $ABC = 40^\circ$, angle $ACB = 55^\circ$ and $AC = AD$.

(a) Calculate AC .

$$\begin{aligned}\frac{AC}{\sin 40^\circ} &= \frac{12.5}{\sin 55^\circ} \\ AC &= \frac{12.5 \sin 40^\circ}{\sin 55^\circ} \\ AC &= 9.808 \\ AC &= 9.81 \text{ cm (3sf)}\end{aligned}$$

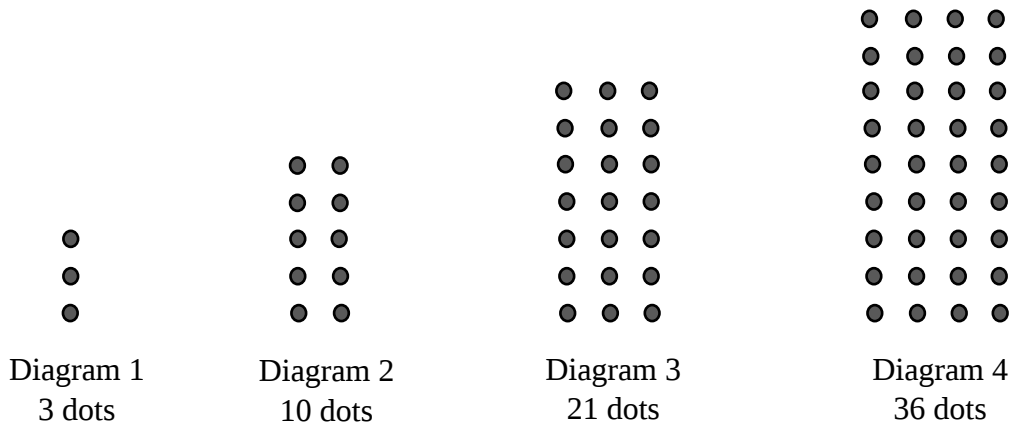
Answer $AC = \dots\dots\dots$ cm [2]

(b) Calculate the area of triangle ADB .

$$\begin{aligned}\triangle ACD \text{ is isosceles with } \angle ACD &= \angle ADC \\ \angle ADC &= 55^\circ \\ \therefore \angle ADB &= 180^\circ - 55^\circ = 125^\circ \text{ (adj } \angle \text{s on a str line)} \\ \therefore \angle DAB &= 180^\circ - 125^\circ - 40^\circ = 15^\circ \text{ (}\angle \text{ sum of } \triangle) \\ \text{or } \therefore \angle DAB &= 55^\circ - 40^\circ = 15^\circ \text{ (ext } \angle \text{ of } \triangle) \\ \text{Area of } \triangle ADB &= \frac{1}{2}(9.808)(12.5)\sin 15^\circ \\ \text{Area of } \triangle ADB &= 15.9 \text{ cm}^2 \text{ (3sf)}\end{aligned}$$

Answer $\dots\dots\dots$ cm^2 [3]

- 18 The first four diagrams in a sequence are shown below.



- (a) Find the number of dots in Diagram 5.

$$5 \times 11 = 55$$

Answer [1]

- (b) Find an expression, in terms of n , for the number of dots in Diagram n .

$$n(2n+1)$$

Answer [2]

- (c) Find the difference in the number of dots between Diagram 7 and Diagram 6.

Diagram 7 has 105 dots
Diagram 6 has 78 dots
difference = $105 - 78 = 27$

Answer [1]

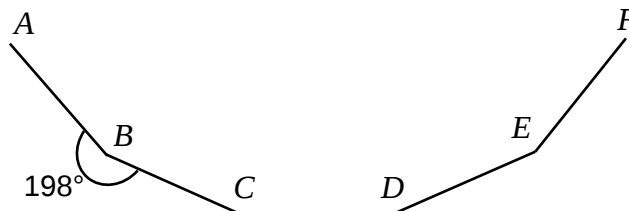
- (d) Show that the difference in the number of dots between Diagram $(n + 1)$ and Diagram n is $4n + 3$.

Answer

$$\begin{aligned} \text{Diagram } (n+1) \text{ has } & (n+1)(2n+3) \\ & (n+1)(2n+3) - n(2n+1) \\ & = 2n^2 + 5n + 3 - 2n^2 - n \\ & = 4n + 3 \end{aligned}$$

[2]

19



The diagram shows part of a regular polygon $ABCDEF....$
 Given that the reflex angle $ABC = 198^\circ$

- (a) Prove that triangle ABC and triangle CDE are congruent.

Answer

$AB = CD$ (sides of regular polygon)
 $BC = DE$ (sides of regular polygon)
 $\angle ABC = \angle CDE$ (interior angle of regular polygon)
 $\triangle ABC \equiv \triangle CDE$ (SAS)

[2]

- (b) Find the size of each exterior angle.

Interior Angle = $360 - 198$ (at a point)
 = 162

or Exterior Angle = $198 - 180$

Exterior Angle = $180 - 162$ (adj on str lrs)
 = 18

= 18

- (c) Find the value of n .

$$\begin{aligned}
 n &= \frac{360^\circ}{18^\circ} \\
 &= 20
 \end{aligned}$$

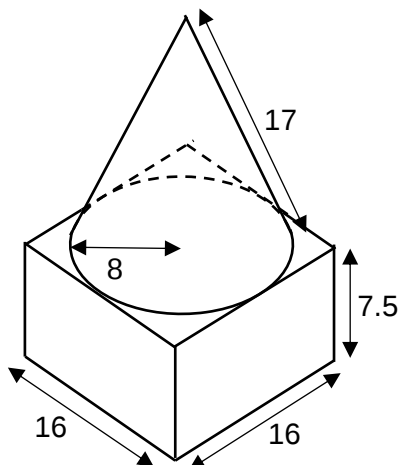
Answer $n = \dots\dots\dots$ [1]

- (d) Find the angle BEF .

$\angle BED = 180^\circ - 162^\circ$ (int $\angle$ $BE \parallel CD$)
 = 18
 $\angle BEF = 162^\circ - 18^\circ$
 = 144

Answer Angle $BEF = \dots\dots\dots$ [2]

20



The diagram shows a solid formed from a cone and a cuboid.
 The cone has a radius of 8 cm and slant height of 17 cm.
 The cuboid has a square base of length 16 cm and height of 7.5 cm.

- (a) Calculate the volume of the solid figure.

$$\begin{aligned} \text{height} &= \sqrt{17^2 - 8^2} \\ &= 15 \text{ cm} \\ \text{Volume} &= \frac{1}{3} \pi (8)^2 (15) + (16 \times 16 \times 7.5) \\ &= 2925.3 \\ &= 2930 \text{ cm}^3 \quad (3\text{sf}) \end{aligned}$$

Answer « « « « « « « « « « cm³ [3]

- (b) Calculate the total surface area of the solid figure.

$$\begin{aligned} \text{Total SA} &= \pi (8)(17) + (16 \times 16) + 4(8 \times 16) + (16 \times 16) + 4(16 \times 7.5) \\ &= 1218.1 \\ &= 1220 \text{ cm}^2 \quad (3\text{sf}) \end{aligned}$$

Answer « « « « « « « « « « cm² [3]

- 21 At bakery A, a chocolate bread loaf costs \$2.50, a wholemeal bread loaf costs \$3 and a raisin bread loaf costs \$3.80 .

At bakery B, a chocolate bread loaf costs \$3.50, a wholemeal bread loaf costs \$3.20 and a raisin bread loaf costs \$3.

This information can be represented by the matrix $P = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} C \\ W \\ R \end{matrix} & \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix} \end{matrix}$

- (a) Janet buys 2 loaves of chocolate bread, 4 loaves of wholemeal bread and 3 loaves of raisin bread.
Karen buys 5 loaves of chocolate bread, 3 loaves of wholemeal bread and x loaves of raisin bread.
Represent this information in a 2×3 matrix, N .

$$N = \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix}$$

Answer N [1]

- (b) Find, in terms of x , the matrix $S = NP$.

$$S = NP$$

$$= \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix} \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix}$$

$$= \begin{pmatrix} 28.4 & 28.8 \\ 21.5 + 3.8x & 27.1 + 3x \end{pmatrix}$$

Answer S [2]

- (c) Karen has a budget of \$30.
By writing an inequality in x , determine the total number of loaves of bread she can buy from bakery B.

For bakery B,

$$27.1 + 3x \leq 30$$

$$3x \leq 2.9$$

$$x \leq 0.966$$

?8 loaves

Answer « « « « « « « « loaves [2]

A speed-time graph for a car starting from rest. The vertical axis is labeled 'Speed (m/s)' and has markings at 0, 12, and 20. The horizontal axis is labeled 'Time, t (seconds)' and has markings at 0, 10, and 22. The graph consists of three segments: a straight line from (0, 0) to (10, 12), a straight line from (10, 12) to (22, 20), and a horizontal line from (22, 20) to the right. Dashed lines connect the points (10, 12) and (22, 20) to their respective values on the axes.

- $$\begin{aligned}\text{acceleration} &= \frac{v-12}{13-10} = \frac{20-12}{22-10} \\ \frac{v-12}{3} &= \frac{8}{12} \\ v-12 &= 2 \\ \therefore v &= 14\end{aligned}$$

P[2]V

- $$\begin{aligned} &\text{distance} \\ &= \frac{1}{2} \times 10 \times 12 + \frac{1}{2} (20 + 12) \times 12 \\ &= 60 + 192 \\ &= 252 \end{aligned}$$

[2]

- (c) A motor cyclist starts 13 seconds later from the same place as the car, and accelerates uniformly for 9 seconds until it reaches a speed of 25 m/s

Is the motor cyclist behind the car at $t = 22$?
Justify your decision

Answer

distance by car = 252 m

$$\text{distance by motor cyclist} = \frac{1}{2} \times 9 \times 25$$

distance by motor cyclist = 112.5 m < 252 m

$\therefore$ motor cyclist is behind the car at $t = 22$

[2]

Time (t minutes)	Frequency
8 to 10	6
10 to 12	25
12 to 14	36
14 to 16	29
16 to 18	4

- $$\begin{aligned}\text{Mean} &= \frac{1300}{100} \\ &= 13\end{aligned}$$

Midpoint	Frequency
9	6
11	25
13	36
15	29
17	4

Answer « « « « « .. « « « « minutes [1]

- (ii) the standard deviation of their timing.

$$SD \sqrt{\frac{17276}{100}} = 13.3$$

Answer « « « « « .. « « « « minutes [1]

- (iii) The meartime for another group of 100 males was 12.6 minutes and the standard deviation was 2.5 minutes.

Make two comparisons between the times for males and the times for females.

Answer «

The males ran faster as the mean (of 6.12 minutes) is lower than the females mean time (8.1 minutes). The timing for the females shows more consistency as its SD (1.94 minutes) is lower as compared to the SD for the males (of 2.5 minutes).

« « « « « « « «
« « « « « « « «
« « « « « « « «
« « « « « « « «

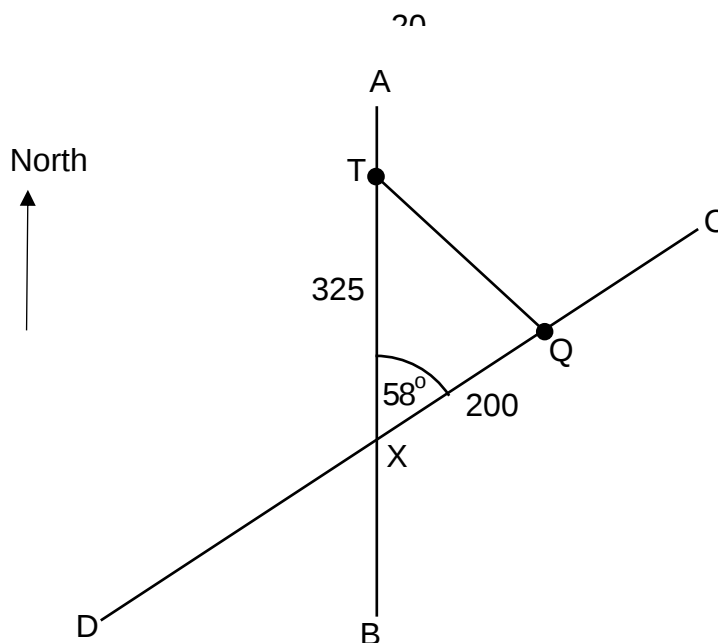
« « « « « « » @

- (b) Two female runners are chosen at random.

Find the probability that both runners complete the 2.4 km run in less than 12 minutes.

$\frac{31}{100}$	$\frac{30}{99}$	$\frac{31}{330}$
------------------	-----------------	------------------

Answer « « « « « .. « « « « « [2]



AB and CD are two roads which meet at point X at an angle of 58° , with A due north of X. Point Q is 200 km from point X.

At 1040, a truck T is 325 km north of point X and is travelling at 75 km/h along the path to reach point Q.

(a) At what time would the truck reach point Q?

$$TQ = \sqrt{325^2 + 200^2 - 2(325)(200)\cos 58^\circ}$$

$$TQ = 277.0117 \text{ km}$$

$$\text{Time taken} = \frac{277.0117}{75} \text{ h}$$

$$= 3.6934 \text{ h}$$

$$= 3 \text{ h } 41.6 \text{ min}$$

$$\therefore \text{ at time 1421 or 1422}$$

Answer « « « « « .. « « « « « 3]

(b) Calculate the bearing of Q from T.

$$\frac{\sin \angle XQT}{325} = \frac{\sin 58^\circ}{277.0117}$$

$$\sin \angle XQT = \frac{325 \sin 58^\circ}{277.0117}$$

$$\angle XQT = 84.24^\circ$$

$$\therefore \text{ bearing of } Q \text{ from } T = 58^\circ + 84.24^\circ$$

$$\therefore \text{ bearing of } Q \text{ from } T = 142.2^\circ (1\text{dp})$$

$$\frac{\sin \angle XTQ}{200} = \frac{\sin 58^\circ}{277.0117}$$

$$\sin \angle XTQ = \frac{200 \sin 58^\circ}{277.0117}$$

$$\angle XTQ = 37.75^\circ$$

$$\therefore \text{ bearing of } Q \text{ from } T = 180^\circ - 37.75^\circ$$

$$\therefore \text{ bearing of } Q \text{ from } T = 142.2^\circ (1\text{dp})$$

« « « « 2]

Name: _____ ()

Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS

4052/02

Paper 2

Friday 18 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

Q1		Q4		Q7	
Q2		Q5		Q8	
Q3		Q6		Q9	

Paper 1	/ 90
Paper 2	/90
Total	/100

This document consists of **22** printed pages.



圣尼各拉女校
CHI J ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

- 1 (a) Solve the inequality $\frac{2x-1}{3} \geq \frac{x+5}{4}$.

Answer [2]

- (b) It is given that $3d = b^2(c+5)$.

- (i) Find d when $b = -2$ and $c = -6.2$.

Answer $d =$ [1]

- (ii) Express b in terms of c and d .

Answer $b =$ [2]

(c) Solve the equation $\frac{x}{2x-1} - \frac{3}{x} = 2$.

Give your solutions correct to 2 decimal places.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [5]

- 2 (a) The speed of light is 3×10^8 m/s.

Find the distance in kilometres travelled by light in 100 nanoseconds, giving your answer in standard form.

[1 nanosecond = 10^{-9} seconds]

Answer km [2]

- (b) The mass of an ant is 5 milligrams.
A cat has a mass of 720 000 times that of the ant.

Calculate the mass of the cat in grams in standard form.

[1 milligram = 10^{-3} gram]

Answer g [2]

- (c) In 2023, the Government increased GST to 8%.
At Budget 2023, the Government announced enhancements to the GST Voucher (GSTV) scheme to help Singaporeans with the GST rate increase.

Mrs Lee intends to buy an item that has GST of \$7.

Find the price of the item with GST.

Answer \$ [2]

(d) June buys an item in Korea costing ₩115000.

The exchange rate between Singapore dollars and Korean Won is \$1 = ₩967.54 .

The same item is on sale in Singapore costing \$100.

How much would June have saved if she bought the item in Singapore?

Answer \$ [2]

3 J is a point $(-2, 7)$ and K is a point $(10, 1)$.

(a) Find the length of JK .

Answer [2]

(b) Find the equation of JK .

Answer [2]

(c) The line with equation $6y - 7x = 16$ intersects the line JK at point L .
Find the coordinates of L .

Answer L (..... ,) [3]

(d) State the ratio $JL : JK$.

Answer : [1]

- 4 Hello Panda is a popular Japanese snack. Its cream-filled biscuits are shaped like panda faces. The biscuits are commonly sold in a hexagonal prism box modelled below.

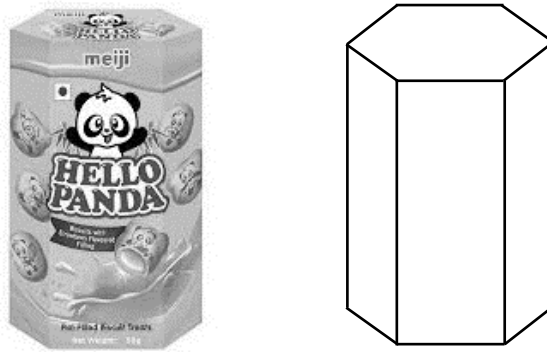


Diagram 1 shows the cross section area of the normal box. It consists of a hexagon formed by two identical trapeziums. $PS = 7.5$ cm, $QR = 4.5$ cm and $PQ = RS = 2.5$ cm.

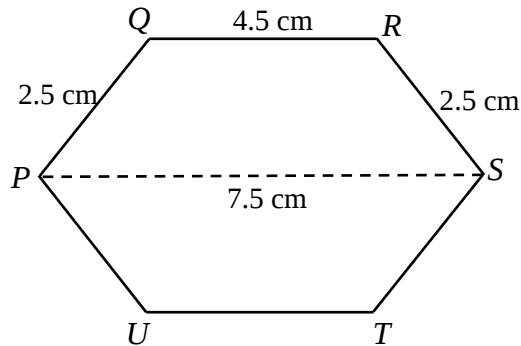


Diagram 1

- (a) Show that the area of trapezium PQRS is 12 cm^2 .

Answer

[2]

- (b) Given that the height of the box is 14 cm, calculate the total surface area of the box.

Answer cm^2 [3]

Hello Panda has a family box, containing ten individual packets. Given that the two boxes are geometrically similar and the height of the family box is 3 times the height of the normal box.

- (c) Calculate the total surface area of the family box, correct to 2 significant figures.

Answer cm^2 [2]

Jack went to the supermarket to purchase some Hello Panda biscuits. They were sold as Normal boxes and Family boxes.

Size	Normal Box	Family Box
Weight	Contains 1 packet weighing 50 grams	Contains 10 packets, each weighing 26 grams
Price	Buy 5 boxes for \$5.80	Buy 1 box for \$6.61

- (d) Which packaging is more value for money? Justify your answer with calculations.

Answer

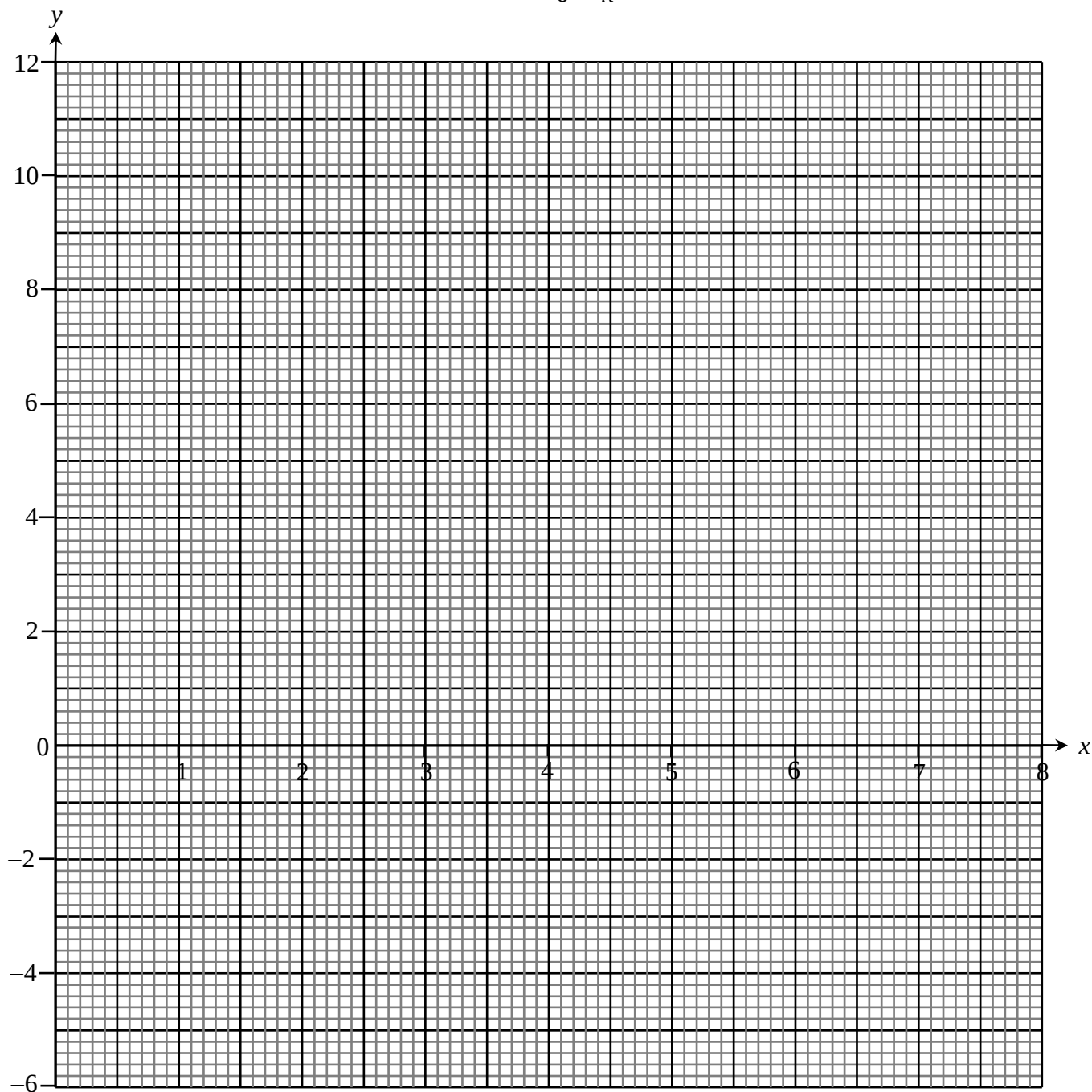
.....
 [3]

- 5 (a) Complete the table of values for $y = \frac{x^2}{6} + \frac{8}{x} - 5$.

x	0.5	1	2	3	4	5	6	7	8
y		3.2	-0.3	-0.8	-0.3	0.8	2.3	4.3	6.7

[1]

- (b) On the grid, draw the graph of $y = \frac{x^2}{6} + \frac{8}{x} - 5$ for $0 < x \leq 8$.



[2]

- (c) The equation $\frac{x^2}{6} + \frac{8}{x} = 8$ has two solutions.

Use your graph to solve the equation.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (d) (i) On the grid in part (b), draw the line $y = 9 - 4x$ for $0 \leq x \leq 8$.

[1]

- (ii) Write down the x -coordinates of points where this line intersects the curve.

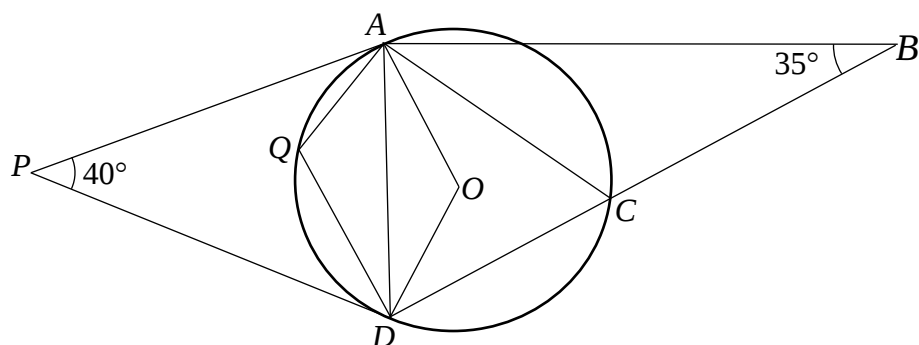
Answer $x = \dots\dots\dots$ and $\dots\dots\dots$ [2]

- (iii) Another line $y = c - 4x$, where c is a constant, intersects the curve at only one point.

Use your graph to find the value of c .

Answer $c = \dots\dots\dots$ [2]

6



The diagram shows a circle $ACDQ$, centre O .

PA and PD are tangents to the circle.

$AC = DC$ and DCB is a straight line.

Angle $APD = 40^\circ$ and angle $ABD = 35^\circ$.

- (a) (i) Find angle AOD .
Give a reason for each step of your working.

Answer Angle $AOD = \dots\dots\dots$ [2]

- (ii) Find angle AQD .
Give a reason for each step of your working.

Answer Angle $AQD = \dots\dots\dots$ [2]

- (iii) Find angle ACD .
Give a reason for each step of your working.

Answer Angle $ACD = \dots\dots\dots$ [1]

- (b) Explain why a semicircle with DB as a diameter, passes through A .

Answer

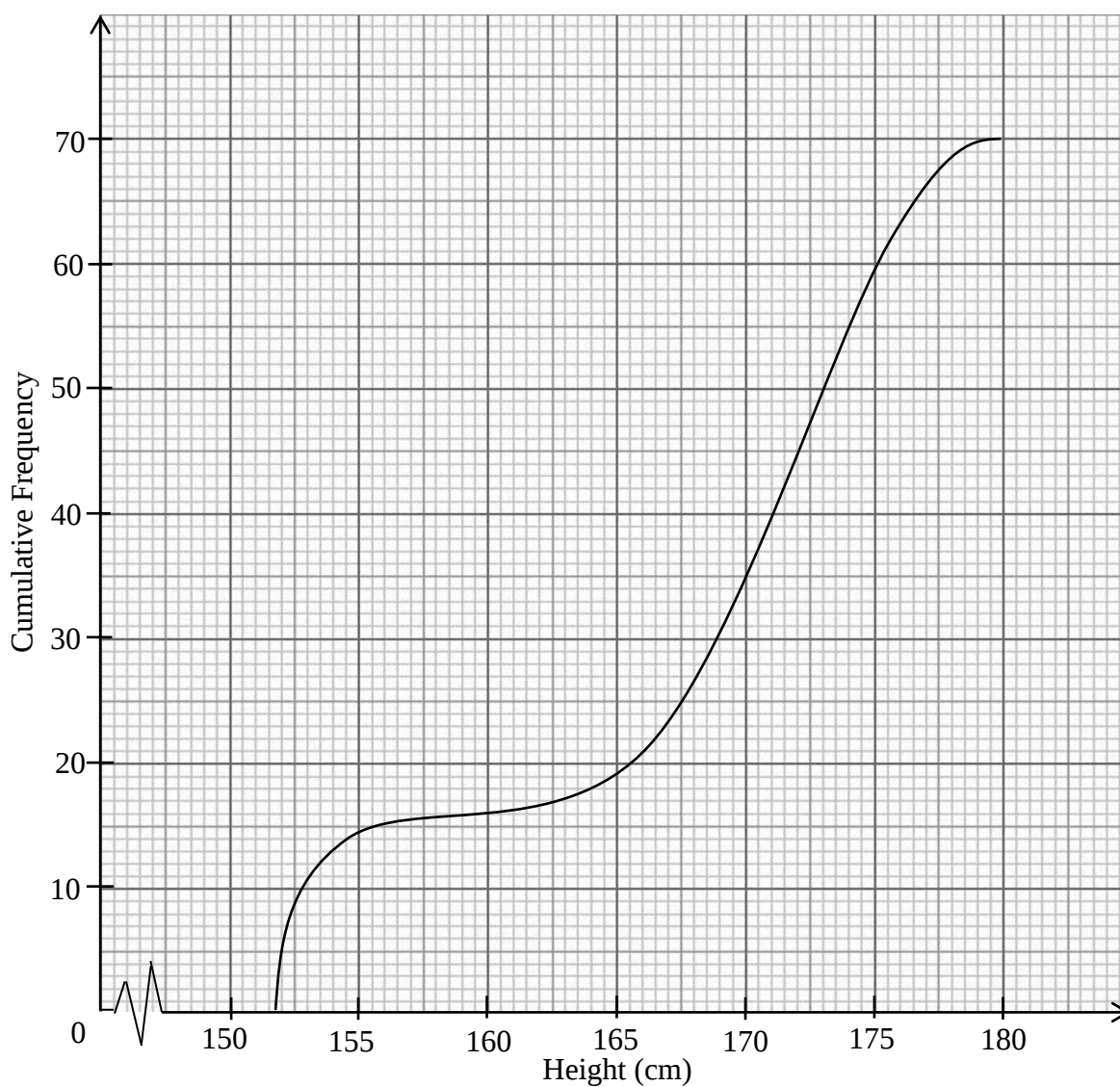
.....
 [2]

- (c) The radius of circle $ACDQ$ is 10 cm.
 Find the area of the region bounded by PA , PD and the minor arc AQD .

Answer cm^2 [4]

- 7 The heights, in cm, of 70 adults are measured.

The cumulative frequency curve shows the distribution of their heights.



- (a) Use the graph to estimate

- (i) the median height,

Answer cm [1]

- (ii) the interquartile range,

Answer cm [2]

- (iii) the percentage of adults who are taller than 174.5 cm.

Answer % [2]

- (b) One adult is selected at random from the 70 adults. If the probability that this adult is shorter than h cm is $\frac{2}{5}$, find the value of h .

Answer $h =$ [2]

- (c) Two adults were selected at random. Find the probability that one of the adults is shorter 154 cm and the other is taller than 176 cm.

Answer [2]

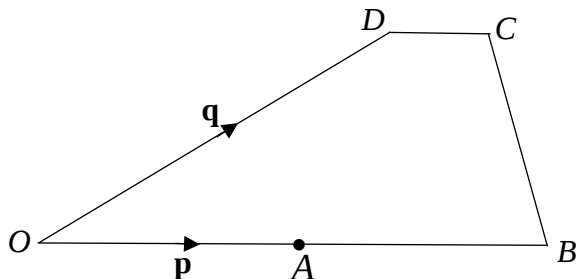
- (d) A mistake has been discovered when measuring the heights of the adults. An additional 2 cm needs to be added to all the heights.

Explain how the median height and the interquartile range have been affected by this mistake.

.....
 [2]

- 8 (a) $OBCD$ is a trapezium where DC is parallel to OB .

$\vec{OA} = \mathbf{p}$ and $\vec{OD} = \mathbf{q}$. $OA = \frac{1}{2}OB$ and $DC : OB = 1 : 6$.



- (i) Show that $\vec{BC} = \mathbf{q} - \frac{5}{3}\mathbf{p}$.

Answer

[2]

- (ii) X is a point on OB such that XD is parallel to BC .
Find $\vec{XC}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer $\vec{XC} = \dots\dots\dots$ [2]

- (iii) Find the ratio area $XBCD$: area $OBCD$.

Answer : [1]

- (iv) Y is a point on BC produced such that $\vec{OY} = h \mathbf{q}$.
By using $\vec{BY} = k\vec{BC}$, find the value of h and the value of k .

Answer $h = \dots\dots\dots$

$k = \dots\dots\dots$ [3]

(b) T is the point $(-2, 6)$ and $\overrightarrow{ST} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

(i) Find the position vector of S .

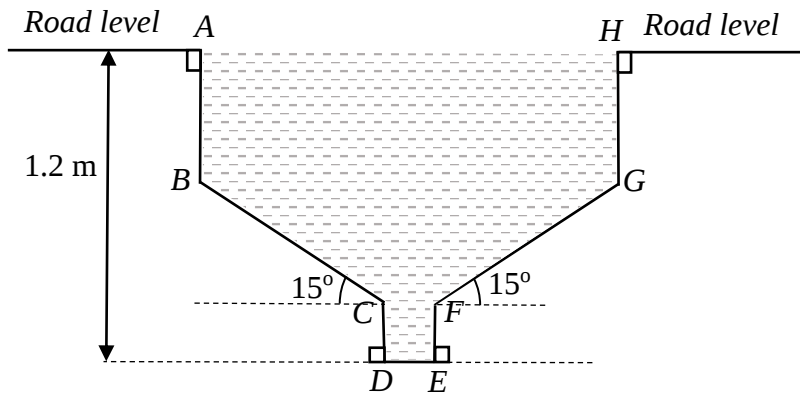
Answer $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(ii) Find the magnitude of $\begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

Answer [1]

- 9 A project manager is constructing a drain running along a straight boundary of a piece of land. The drain runs along a slope with a bed gradient of $\frac{1}{200}$ which means for every 200 m length there is a drop of 1 m.

The diagram below, not drawn to scale, shows the cross section of the drain.



The depth of the drain is 1.2 metre from the road level, with $AB = HG$.

$BC = FG = 27$ cm, $CD = EF = 10$ cm and $DE = 4$ cm.

The wetted perimeter of the planned drain is the sum of the length of AB , BC , CD , DE , EF , FG and GH .

- (a) Show that the cross-sectional area $ABCDEFGH$ of the planned drain is 0.6035 m^2 , correct to 4 decimal places.

Answer

[4]

The velocity of water flow is computed from the Manning's Formula:

$$v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}},$$

where v = velocity of water flow in the drain (m/s)

n = roughness coefficient

A = flow area* (m²)

P = wetted perimeter (m)

$R = \frac{A}{P}$ = hydraulic radius (m)

S = bed gradient

*Flow area is the cross-sectional area of the drain.

The value of the roughness coefficient (n) depends on the drain's flow surface and is given below.

Material used to construct the drain	Roughness coefficient (n)
Unplasticised Polyvinyl Chloride (PVC)	0.0125
Concrete	0.0150
Brick	0.0170
Earth	0.0270
Mud	0.0350
Gravel	0.0300

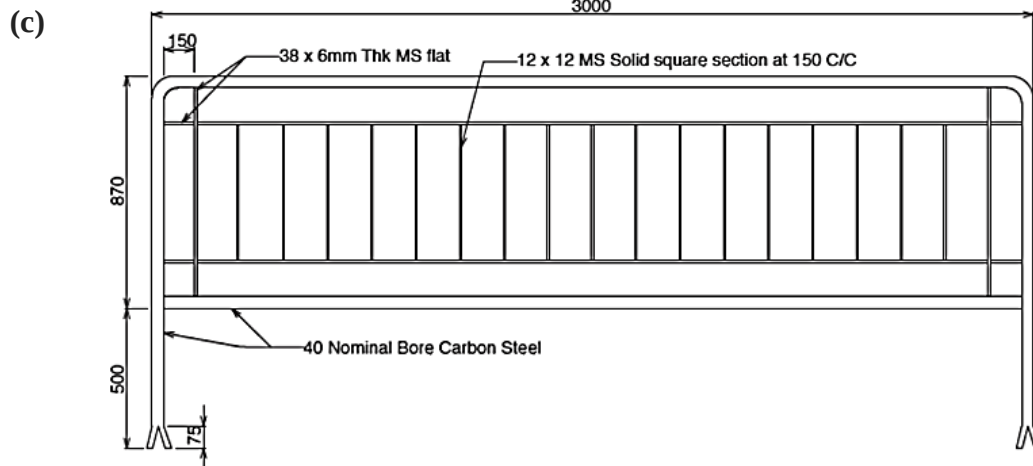
The safety requirements of a drain are listed below.

- The minimum velocity of water flow in a drain must exceed 3.6 km/h.
- The maximum velocity of water flow in a mud drain must be no more than 5.4 km/h.
- The maximum velocity of water flow in any other drain must be no more than 10.8 km/h.

- (b) Concrete is used as a material for the construction of the drain. Suggest whether the safety requirements will be met. Justify your decision and show your calculations clearly.

Answer

[6]



This diagram shows one standard safety railing taken from <https://kusgrp.com/metal-works/product/mild-steel-railing/>

The open drain runs for the length of 47 metres.

For open drains of more than 1.0 m deep, standard safety railings are placed along the side of the drain.

A standard safety railing measures 3000 mm by 1370 mm, with a 50 mm space in between each railing.

Assuming the first railing is placed at the start of the drain, the project manager claims that there is a need to modify the last railing to sufficiently line the open drain safely.

Calculate the length, in mm, of the last railing.

Answer mm [2]

Answer 2023 MA Prelim Paper 2

1. (a) $x \geq 3.8$
(b) (i) $d = -1.6$
(ii) $b = \pm \sqrt{\frac{3d}{c+5}}$
(c) $x = 0.54, -1.87$
2. (a) 3.0×10^{-2}
(b) 3.6×10^3
(c) \$94.50
(d) \$18.86
3. (a) 13.4
(b) $y = -\frac{1}{2}x + 6$
(c) (2, 5)
(d) 1:3
4. (a) show
(b) 314 cm^2
(c) 2800 cm^2
(d) show $0.0232 \text{ g/g} < 0.0254 \text{ g/g}$ or $43.1 \text{ g/g} > 39.3 \text{ g/g}$ with concluding statement
5. (a) 11.0
(b) plot and draw graph
(c) show $y = 3$ in working and draw on graph to find $x = 1, 6.4$
(d) (i) draw line on grid
(ii) $x = 0.7, 2.4$
(iii) draw tangent with gradient -4 and read the y-intercept. $c = 6.6$
6. (a) (i) 140°
(ii) 110°
(iii) 70°
(b) use right angle in semicircle or angle at centre = $2 \times$ angle at circumference
(c) 153 cm^2
7. (a) (i) 170 cm
(ii) 10 cm
(iii) 18.6%
(b) 168.5
(c) $\frac{13}{345}$
(d) with the mistake, the median height has to add 2 cm to get the actual median height as 172 cm but the interquartile range which is the difference between the upper and lower quartiles remain unchanged.

8. (a) (i) start with vector equation of BC

(ii) $q - \frac{4}{3}p$

(iii) 2:7

(iv) $h = k = 1.2$

$$\overrightarrow{OS} = \overrightarrow{OT} + \overrightarrow{TS}$$

(b) (i) $= \begin{pmatrix} -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -7 \\ 9 \end{pmatrix}$
 $= \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1

(ii) 11.4

9. (a) show

(b) $v = 1.68 \text{ m/s}$ or 6.04 km/h

$1 \text{ m/s} < 1.68 \text{ m/s} < 3 \text{ m/s}$ or $3.6 \text{ km/h} < 6.04 \text{ km/h} < 10.8 \text{ km/h}$

Yes, safety requirement is met

(c) 1250 mm

Name: _____ ()

Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS **Student Copy**

4052/02

Paper 2

Friday 18 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

Q1		Q4		Q7	
Q2		Q5		Q8	
Q3		Q6		Q9	

Paper 1	/ 90
Paper 2	/90
Total	/100

This document consists of **22 printed pages**.



圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

- 1 (a) Solve the inequality $\frac{2x-1}{3} \geq \frac{x+5}{4}$.

$$\begin{aligned}\frac{2x-1}{3} &\geq \frac{x+5}{4} \\ 4(2x-1) &\geq 3(x+5) \\ 8x-4 &\geq 3x+15 \\ 5x &\geq 19 \\ x &\geq 3.8 \text{ or } 3\frac{4}{5} \text{ or } \frac{19}{5}\end{aligned}$$

Answer [2]

- (b) It is given that $3d = b^2(c+5)$.

- (i) Find d when $b = -2$ and $c = -6.2$.

$$\begin{aligned}d &= \frac{(-2)^2(-6.2+5)}{3} \\ d &= -1.6\end{aligned}$$

Answer $d =$ [1]

- (ii) Express b in terms of c and d .

$$\begin{aligned}3d &= b^2(c+5) \\ b^2 &= \frac{3d}{c+5} \\ b &= \pm \sqrt{\frac{3d}{c+5}}\end{aligned}$$

Answer $b =$ [2]

(c) Solve the equation $\frac{x}{2x-1} - \frac{3}{x} = 2$.

Give your solutions correct to 2 decimal places.

$$\frac{x}{2x-1} - \frac{3}{x} = 2$$

$$\frac{x^2 - 3(2x-1)}{x(2x-1)} = 2$$

$$x^2 - 3(2x-1) = 2x(2x-1)$$

$$x^2 - 6x + 3 = 4x^2 - 2x$$

$$3x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{52}}{6}$$

$$x = 0.54, -1.87 \text{ (2dp)}$$

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [5]

- 2 (a) The speed of light is 3×10^8 m/s.

Find the distance in kilometres travelled by light in 100 nanoseconds, giving your answer in standard form.

[1 nanosecond = 10^{-9} seconds]

$$\begin{aligned} \text{Distance} &= \frac{3 \times 10^8}{1000} \times 10^{-9} \times 100 \\ &= 3 \times 10^{-2} \text{ km (standard form)} \end{aligned}$$

Answer km [2]

- (b) The mass of an ant is 5 milligrams.
A cat has a mass of 720 000 times that of the ant.

Calculate the mass of the cat in grams in standard form.

[1 milligram = 10^{-3} gram]

$$\begin{aligned} \text{Mass of cat} &= \frac{720000 \times 5}{1000} \text{ g} \\ &= 3600 \text{ g} \\ &= 3.6 \times 10^3 \text{ g (standard form) A1} \end{aligned}$$

Answer g [2]

- (c) In 2023, the Government increased GST to 8%.
At Budget 2023, the Government announced enhancements to the GST Voucher (GSTV) scheme to help Singaporeans with the GST rate increase.

Mrs Lee intends to buy an item that has GST of \$7.

Find the price of the item with GST.

$$\begin{aligned} \text{price} &= \frac{7}{8} \times 108 \\ &= \$94.50 \end{aligned}$$

Answer \$ [2]

- (d) June buys an item in Korea costing ₩115000.

The exchange rate between Singapore dollars and Korean Won is \$1 = ₩967.54 .

The same item is on sale in Singapore costing \$100.

How much would June have saved if she bought the item in Singapore?

$$\begin{aligned}\text{Cost June paid in total} &= \frac{115000}{967.54} \\ &\approx \$118.858\end{aligned}$$

$$\text{Savings} \approx 118.858 - 100 = \$18.86$$

or

$$\$100 = \text{W } 96754$$

$$\text{Save} = \text{W } 115000 - 96754$$

$$= \text{W } 18246$$

$$= \$ \frac{18246}{967.54}$$

$$= \$18.86$$

Answer \$ [2]

- 3 J is a point $(-2, 7)$ and K is a point $(10, 1)$.

(a) Find the length of JK .

$$\begin{aligned}\text{Distance} &= \sqrt{(-2-10)^2 + (7-1)^2} \\ &= 13.41 \\ &\approx 13.4 \text{ units (3sf)}\end{aligned}$$

Answer [2]

(b) Find the equation of JK .

$$\begin{aligned}m_{JK} &= \frac{7-1}{-2-10} & 1 &= -\frac{1}{2}(10) + c \\ &= -\frac{1}{2} & c &= 6 \\ \therefore y &= -\frac{1}{2}x + 6\end{aligned}$$

Answer [2]

(c) The line with equation $6y - 7x = 16$ intersects the line JK at point L .
Find the coordinates of L .

$$\begin{aligned}y &= -\frac{1}{2}x + 6 & \text{-----(1)} \\ 6y - 7x &= 16 & \text{-----(2)} \\ \text{sub (1) into (2)} \\ 6\left(-\frac{1}{2}x + 6\right) - 7x &= 16 \\ -3x + 36 - 7x &= 16 \\ -10x &= -20 \\ x &= 2 \\ y &= 5 \\ \therefore L(2, 5)\end{aligned}$$

Answer L (.....,) [3]

(d) State the ratio $JL : JK$.

$$\begin{aligned}JL &= \sqrt{(-2-2)^2 + (7-5)^2} = \sqrt{20} \\ JK &= \sqrt{180} \\ \therefore JL : JK &= \sqrt{20} : \sqrt{180} \\ \text{or } JL_{run} : JK_{run} &= 4 : 12 \text{ or } JL_{rise} : JK_{rise} = 2 : 6 \\ \therefore JL : JK &= 1 : 3\end{aligned}$$

- 4 Hello Panda is a popular Japanese snack. Its cream-filled biscuits are shaped like panda faces. The biscuits are commonly sold in a hexagonal prism box modelled below.

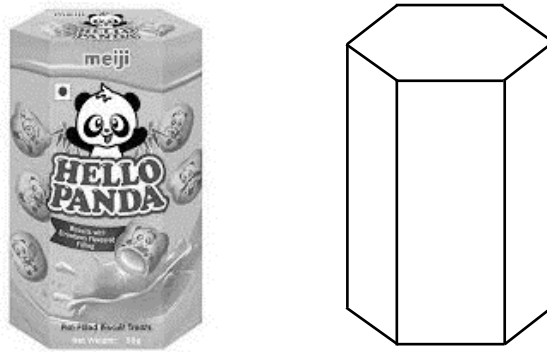


Diagram 1 shows the cross section area of the normal box. It consists of a hexagon formed by two identical trapeziums. $PS = 7.5$ cm, $QR = 4.5$ cm and $PQ = RS = 2.5$ cm.

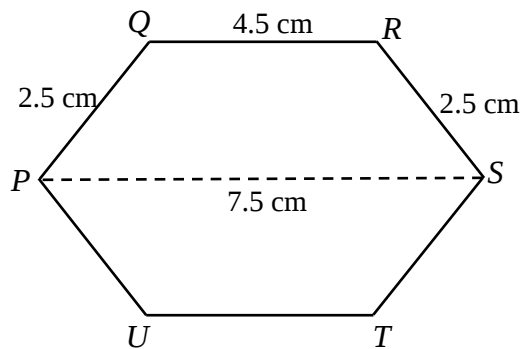
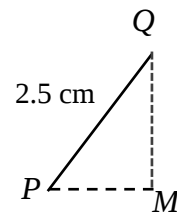


Diagram 1

- (a) Show that the area of trapezium $PQRS$ is 12 cm^2 .

Answer

$$\begin{aligned} PM &= \frac{7.5 - 4.5}{2} = 1.5 \\ QM &= \sqrt{2.5^2 - 1.5^2} = 2 \\ \text{Area} &= \frac{1}{2} \times [4.5 + 7.5] \times 2 \\ &= 12 \text{ cm}^2 \text{ (shown)} \end{aligned}$$



[2]

- (b) Given that the height of the box is 14 cm, calculate the total surface area of the box.

$$\begin{aligned} \text{Total SA} &= (2 \times 12 \times 2) + 4(2.5 \times 14) + 2(4.5 \times 14) \\ &= 314 \text{ cm}^2 \end{aligned}$$

Answer cm^2 [3]

Hello Panda has a family box, containing ten individual packets. Given that the two boxes are geometrically similar and the height of the family box is 3 times the height of the normal box.

- (c) Calculate the total surface area of the family box, correct to 2 significant figures.

$$\begin{aligned}\frac{\text{Area}}{314} &= \left(\frac{3}{1}\right)^2 \\ \text{Area} &= 9 \times 314 \\ &= 2826 \\ &\approx 2800 \text{ cm}^2\end{aligned}$$

Answer cm² [2]

Jack went to the supermarket to purchase some Hello Panda biscuits. They were sold as Normal boxes and Family boxes.

Size	Normal Box	Family Box
Weight	Contains 1 packet weighing 50 grams	Contains 10 packets, each weighing 26 grams
Price	Buy 5 boxes for \$5.80	Buy 1 box for \$6.61

- (d) Which packaging is more value for money? Justify your answer with calculations.

Answer

Normal	Family
$\frac{5.80}{5 \times 50} = 0.0232 \text{ \$/g}$	$\frac{6.61}{10 \times 26} = 0.0254 \text{ \$/g}$
or	
$\frac{5 \times 50}{5.80} = 43.1 \text{ g/\$}$	$\frac{10 \times 26}{6.61} = 39.3 \text{ g/\$}$
Either show $0.0232 < 0.0254$ or $43.1 > 39.3$	
Buying the normal pack (at 5 for \$5.80) is more value for money.	

.....
 [3]

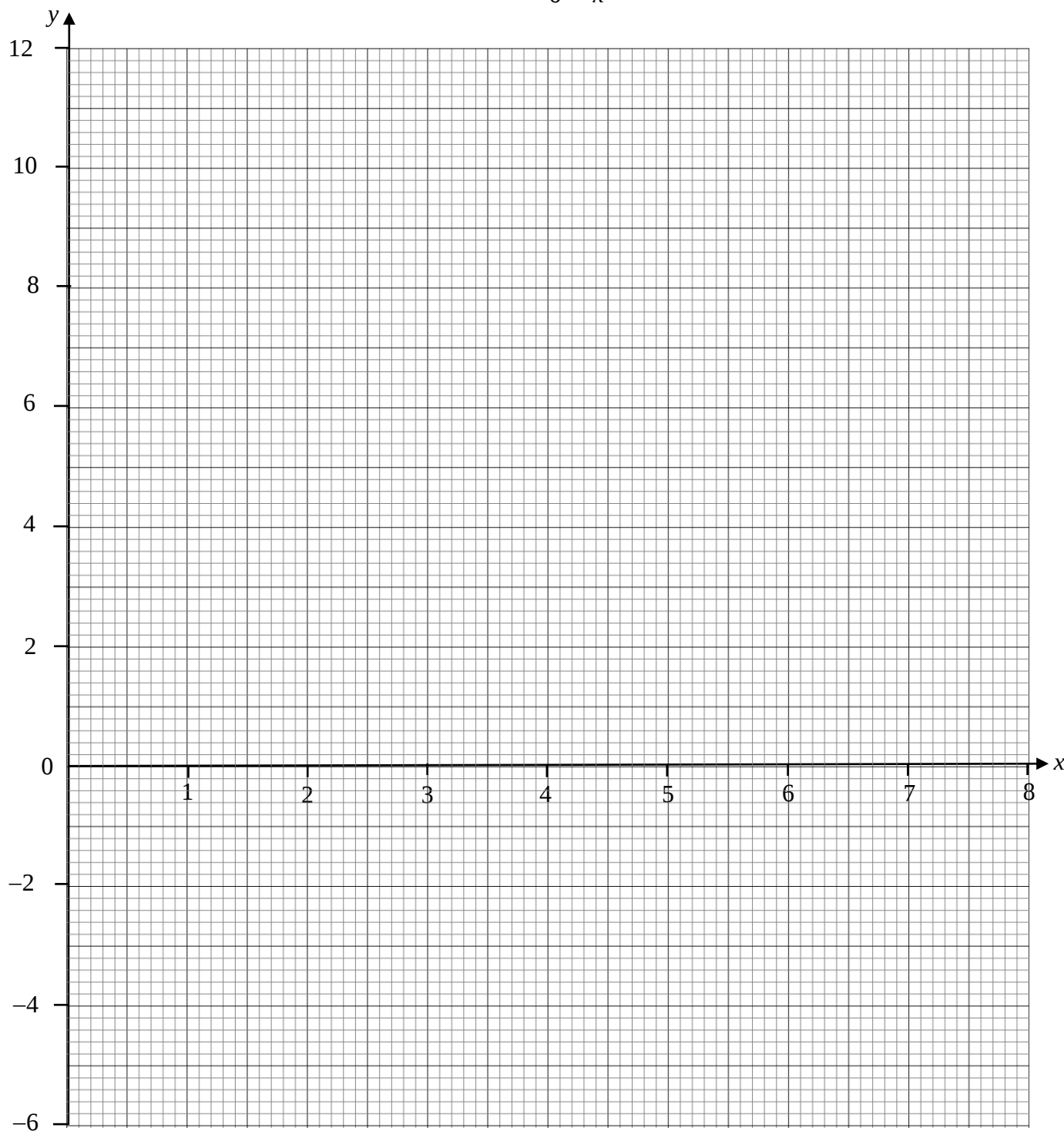
- 5 (a) Complete the table of values for $y = \frac{x^2}{6} + \frac{8}{x} - 5$.

x	0.5	1	2	3	4	5	6	7	8
y		3.2	-0.3	-0.8	-0.3	0.8	2.3	4.3	6.7

Sub $x = 0.5, y = 11.0$

[1]

- (b) On the grid, draw the graph of $y = \frac{x^2}{6} + \frac{8}{x} - 5$ for $0 < x \leq 8$.



[2]

- (c) The equation $\frac{x^2}{6} + \frac{8}{x} = 8$ has two solutions.

Use your graph to solve the equation.

$$\frac{x^2}{6} + \frac{8}{x} - 5 = 8 - 5$$

$$\frac{x^2}{6} + \frac{8}{x} - 5 = 3$$

draw $y = 3$

from graph, $x = 1 \pm 0.1, 6.4 \pm 0.1$

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (d) (i) On the grid in part (b), draw the $4x$ for $0 \leq x \leq 8$.

[1]

- (ii) Write down the x -coordinates of points where this line intersects the line $y = 9 -$
curve.

From graph,
 $x = 0.7 \pm 0.1, 2.4 \pm 0.1$

Answer $x = \dots\dots\dots$ and $\dots\dots\dots$ [2]

- (iii) Another line $y = c - 4x$, where c is a constant, intersects the curve at only one point.

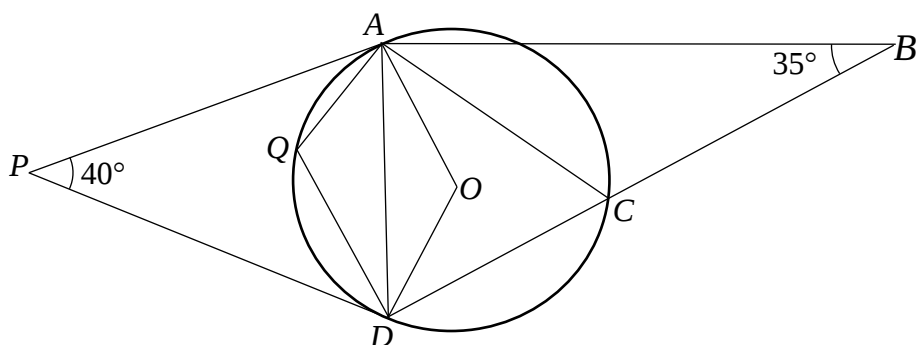
Use your graph to find the value of c .

Draw tangent
From graph,
 $y = 6.6 \pm 0.2$

Answer $c = \dots\dots\dots$ [2]



6



The diagram shows a circle $ACDQ$, centre O .

PA and PD are tangents to the circle.

$AC = DC$ and DCB is a straight line.

Angle $APD = 40^\circ$ and angle $ABD = 35^\circ$.

- (a) (i) Find angle AOD .
Give a reason for each step of your working.

$$\begin{aligned}\angle PAO &= \angle PDO = 90^\circ \text{ (tangent } \perp \text{ radius)} \\ \angle AOD &= 360^\circ - 40^\circ - 90^\circ - 90^\circ \text{ (}\angle \text{sum of quad)} \\ &= 140^\circ\end{aligned}$$

Answer Angle $AOD = \dots\dots\dots$ [2]

- (ii) Find angle AQD .
Give a reason for each step of your working.

$$\begin{aligned}\text{Reflex } \angle AOD &= 360^\circ - 140^\circ \text{ (}\angle \text{s at a pt)} \\ &= 220^\circ \\ \angle AQD &= \frac{220^\circ}{2} \text{ (}\angle \text{at centre} = 2\angle \text{at circumf)} \\ &= 110^\circ\end{aligned}$$

Answer Angle $AQD = \dots\dots\dots$ [2]

- (iii) Find angle ACD .
Give a reason for each step of your working.

$$\begin{aligned}\angle ACD &= \frac{140^\circ}{2} = 70^\circ \text{ (}\angle \text{at centre} = 2\angle \text{at circumf)} \\ \text{OR } \angle ACD &= 180^\circ - 110^\circ \text{ (}\angle \text{s in opp segment)} \\ &= 70^\circ\end{aligned}$$

Answer Angle $ACD = \dots\dots\dots$ [1]

- (b) Explain why a semicircle with DB as a diameter, passes through A .

Answer

Since $AC=DC$ (given), base $\angle$ s of isos Δ

$$\angle ADC = \angle DAC = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$\angle CAB = 70^\circ - 35^\circ = 35^\circ \text{ (ext } \angle \text{ of } \Delta)$$

$$\angle DAB = 55^\circ + 35^\circ = 90^\circ$$

Hence by **right angle in semicircle** property, a semicircle with DB as diameter will pass through A .

.....
 [2]

- (c) The radius of circle $ACDQ$ is 10 cm.
 Find the area of the region bounded by PA , PD and the minor arc AQD .

$$\angle APO = \frac{40^\circ}{2} = 20^\circ \text{ (tangents from external pt)}$$

$$\tan \angle APO = \frac{10}{AP}$$

$$AP = \frac{10}{\tan 20^\circ} \text{ or } 10 \tan 70^\circ$$

$$= 27.47477$$

$$\text{Area } AODP = 2 \left[\frac{1}{2} (27.47477)(10) \right]$$

$$\approx 274.7477$$

$$\text{Area minor sector } ODQA = \frac{140^\circ}{360^\circ} \times \pi \times 10^2$$

$$\text{Area of sector} = 122.17304 \text{ cm}^2$$

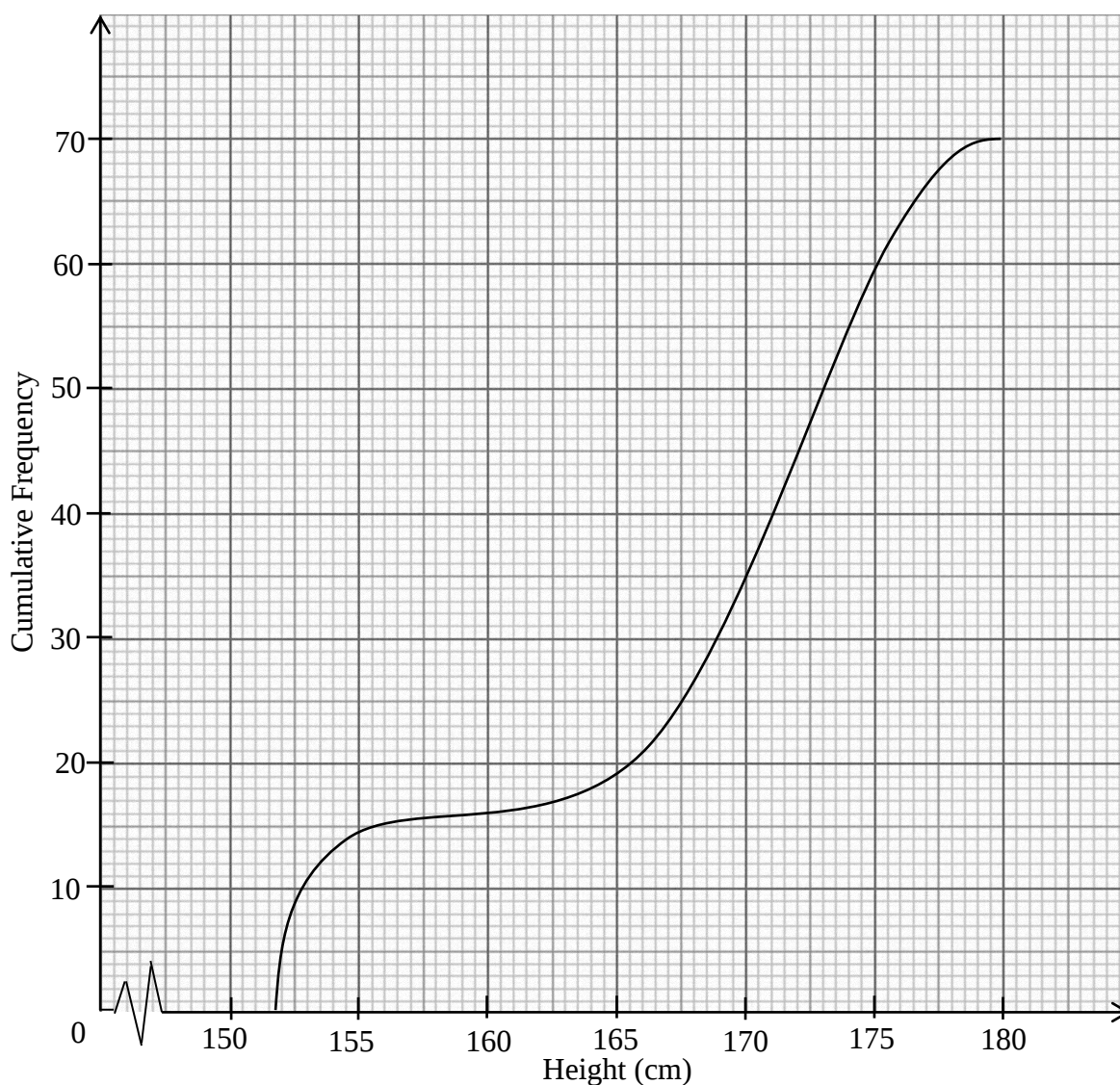
$$\text{Required area} = 274.7477 - 122.1730$$

$$= 153 \text{ cm}^2 \text{ (3sf)}$$

Answer cm^2 [4]

- 7 The heights, in cm, of 70 adults are measured.

The cumulative frequency curve shows the distribution of their heights.



- (a) Use the graph to estimate

- (i) the median height,

Median height = 170 cm

Answer cm [1]

- (ii) the interquartile range,

$$\begin{aligned} Q_1 &= 163.5 \quad (17.5\text{th}) & Q_3 &= 173.5 \quad (52.5\text{th}) \\ \text{ITQ} &= 173.5 - 163.5 \\ &= 10 \end{aligned}$$

Answer cm [2]

- (iii) the percentage of adults who are taller than 174.5 cm.

$$\text{Percentage} = \frac{70 - 57}{70} \times 100\%$$

$$\approx 18.6\% \text{ (3sf)}$$

Answer % [2]

- (b) One adult is selected at random from the 70 adults. If the probability that this adult is shorter than h cm is $\frac{2}{5}$, find the value of h .

$$\frac{2}{5} \text{ represents 28 adults}$$

$$h = 168.5$$

Answer $h =$ [2]

- (c) Two adults were selected at random. Find the probability that one of the adults is shorter 154 cm and the other is taller than 176 cm.

$$\text{Probability} = \frac{13}{70} \times \frac{7}{69} \times 2$$

$$= \frac{13}{345}$$

Answer [2]

- (d) A mistake has been discovered when measuring the heights of the adults. An additional 2 cm needs to be added to all the heights.

Explain how the median height and the interquartile range have been affected by this mistake.

$$\text{New Median height} = 170 + 2$$

$$= 172 \text{ cm}$$

$$\text{New Interquartile Range} = 10 \text{ cm}$$

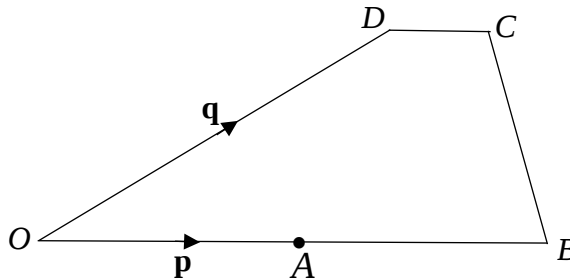
With the mistake, the median height has to add 2 cm more, but the interquartile range which is the difference between the upper and lower quartiles remain unchanged.

.....

..... [2]

- 8 (a) $OBCD$ is a trapezium where DC is parallel to OB .

$\vec{OA} = \underline{p}$ and $\vec{OD} = \underline{q}$. $OA = \frac{1}{2}OB$ and $DC : OB = 1 : 6$.



- (i) Show that $\vec{BC} = \underline{q} - \frac{5}{3}\underline{p}$.

Answer

$$\begin{aligned}\vec{BC} &= \vec{OC} - \vec{OB} \\ &= \underline{q} + \frac{1}{6}(2\underline{p}) - 2\underline{p} \\ &= \underline{q} - \frac{5}{3}\underline{p}\end{aligned}$$

[2]

- (ii) X is a point on OB such that XD is parallel to BC .
Find $\vec{XC}$, as simply as possible, in terms of $\underline{p}$ and $\underline{q}$.

$$\begin{aligned}\vec{XC} &= \vec{XB} + \vec{BC} \\ &= \vec{DC} + \vec{BC} \\ &= \frac{1}{6}(2\underline{p}) + \underline{q} - \frac{5}{3}\underline{p} \\ &= \frac{1}{3}\underline{p} + \underline{q} - \frac{5}{3}\underline{p} \\ &= \underline{q} - \frac{4}{3}\underline{p}\end{aligned}$$

Answer $\vec{XC} = \dots\dots\dots$ [2]

- (iii) Find the ratio area $XBCD$: area $OBCD$.

area $XBCD$: area $OBCD$

$$\begin{aligned}
 &= XB \times h : \frac{1}{2}(DC + OB)h \\
 &= \frac{1}{3} : \frac{1}{2}\left(2 + \frac{1}{3}\right) &= XB : \frac{1}{2}(DC + OB) \\
 &= 2 : 3\left(\frac{7}{3}\right) &= 1 : \frac{1}{2}(1 + 6) \\
 &= 2 : 7 &= 1 : \frac{7}{2} \\
 & &= 2 : 7
 \end{aligned}$$

Answer : [1]

- (iv) Y is a point on BC produced such that $\overrightarrow{OY} = h \mathbf{q}$.
By using $\overrightarrow{BY} = k\overrightarrow{BC}$, find the value of h and the value of k .

$$\overrightarrow{OY} = \overrightarrow{OB} + \overrightarrow{BY}$$

$$h\mathbf{q} = 2\mathbf{p} + k\mathbf{q} - \frac{5k}{3}\mathbf{p}$$

$$h\mathbf{q} = \left(2 - \frac{5k}{3}\right)\mathbf{p} + k\mathbf{q}$$

$$h = k, \quad 2 - \frac{5k}{3} = 0$$

$$h = \frac{6}{5} = 1\frac{1}{5}, \quad k = \frac{6}{5} = 1\frac{1}{5}$$

Answer $h =$

$k =$ [3]

(b) T is the point $(-2, 6)$ and $\overrightarrow{ST} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

(i) Find the position vector of S .

$$\begin{aligned}\overrightarrow{OT} &= \begin{pmatrix} -2 \\ 6 \end{pmatrix} \\ \overrightarrow{OS} &= \overrightarrow{OT} + \overrightarrow{TS} \\ &= \begin{pmatrix} -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -7 \\ 9 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ -3 \end{pmatrix} \\ A &= (5, -3)\end{aligned}$$

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [1]

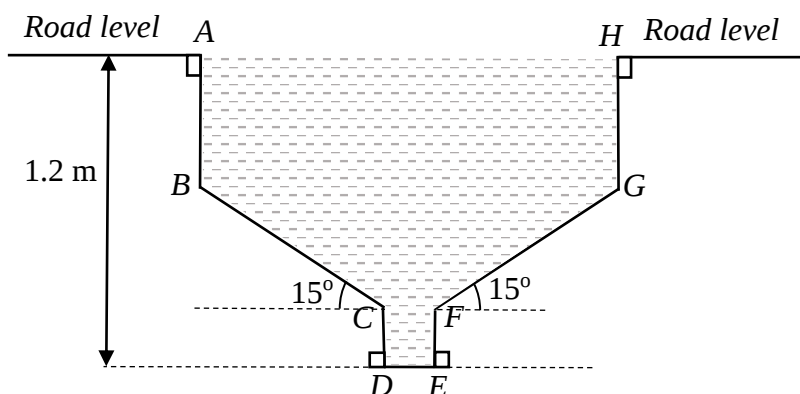
(ii) Find the magnitude of $\begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

$$\begin{aligned}\text{Magnitude} &= \sqrt{(-7)^2 + 9^2} \\ &= \sqrt{130} \approx 11.401 = 11.4 \text{ units (3 s.f.)}\end{aligned}$$

Answer [1]

- 9 A project manager is constructing a drain running along a straight boundary of a piece of land. The drain runs along a slope with a bed gradient of $\frac{1}{200}$ which means for every 200 m length there is a drop of 1 m.

The diagram below, not drawn to scale, shows the cross section of the drain.



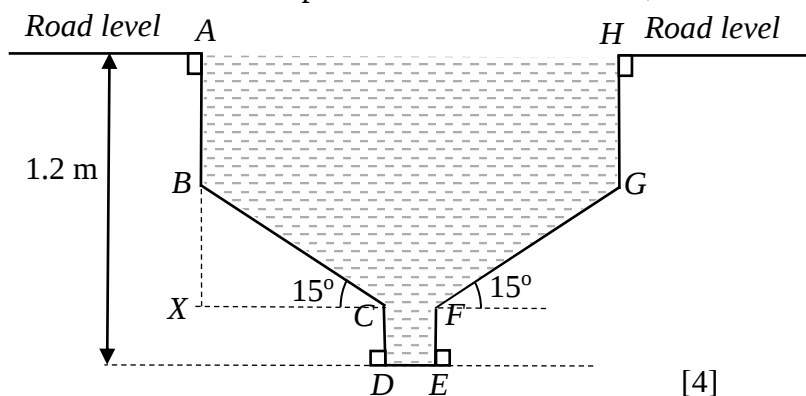
The depth of the drain is 1.2 metre from the road level, with $AB = HG$.

$BC = FG = 27$ cm, $CD = EF = 10$ cm and $DE = 4$ cm.

The wetted perimeter of the planned drain is the sum of the length of AB , BC , CD , DE , EF , FG and GH .

- (a) Show that the cross-sectional area $ABCDEFGH$ of the planned drain is 0.6035 m^2 , correct to 4 decimal places.

Answer



[4]

	Lengths	Area (method 1)		Area (method 2)	
1	CD = 10 DE = 4	40	☐	480	☐
2	$CX = 27\cos 15^\circ = 26.0799$ $BX = 27\sin 15^\circ = 6.9881$	$\frac{1}{2}(CX)(BX)2 + 4BX$ $= 210.2013$		$\frac{1}{2}(CX)(BX)2$ $= 182.2489$	
3	$AB = 120 - (27\sin 15^\circ + 10)$ $= 103.0118$	$(AB)(CX)2 + 4AB$ $= 5373.0748 + 412.0472$ $= 5785.122$		$(AB)(CX)2$ $= 5373.0748$	
	in cm^2	6035.3233		6035.3237	
	in m^2 (see divide 10000)	0.6035		0.6035	

The velocity of water flow is computed from the Manning's Formula:

$$v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}},$$

where v = velocity of water flow in the drain (m/s)

n = roughness coefficient

A = flow area* (m²)

P = wetted perimeter (m)

$R = \frac{A}{P}$ = hydraulic radius (m)

S = bed gradient

*Flow area is the cross-sectional area of the drain.

The value of the roughness coefficient (n) depends on the drain's flow surface and is given below.

Material used to construct the drain	Roughness coefficient (n)
Unplasticised Polyvinyl Chloride (PVC)	0.0125
Concrete	0.0150
Brick	0.0170
Earth	0.0270
Mud	0.0350
Gravel	0.0300

The safety requirements of a drain are listed below.

- The minimum velocity of water flow in a drain must exceed 3.6 km/h.
- The maximum velocity of water flow in a mud drain must be no more than 5.4 km/h.
- The maximum velocity of water flow in any other drain must be no more than 10.8 km/h.

- (b) Concrete is used as a material for the construction of the drain.
Suggest whether the safety requirements will be met.
Justify your decision and show your calculations clearly.

Answer

$$AB = HG = 120 - (27 \sin 15^\circ) - 10$$

$$AB = HG = 103.011 \text{ cm}$$

wetted perimeter

$$= 2(103.011 + 27 + 10) + 4$$

$$= 284.022 \text{ cm}$$

$$= 2.84022 \text{ m}$$

$$R = \frac{A}{P} = \frac{6035.323}{284.022} \text{ or } \frac{0.6035323}{2.84022}$$

$$\therefore R = 21.249 \text{ cm or } 0.21249 \text{ m}$$

$$S = \frac{1}{200}$$

$$n = 0.0150$$

$$v = \left(\frac{1}{0.0150} \right) (0.21249)^{\frac{2}{3}} \left(\frac{1}{200} \right)^{\frac{1}{2}}$$

$$v = 1.678 \text{ m/s}$$

$$v = \frac{1.678 \times 3600}{1000} \text{ km/h}$$

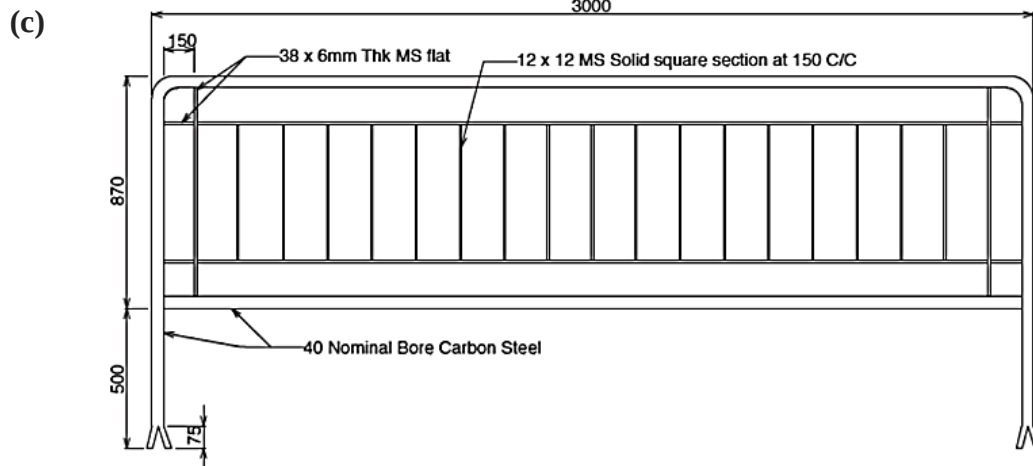
$$v = 6.0408 = 6.04 \text{ km/h (3sf)}$$

$$\therefore 3.6 \text{ km/h} < 6.04 \text{ km/h} < 10.8 \text{ km/h}$$

$$\text{or } 1 \text{ m/s} < 1.678 \text{ m/s} < 3 \text{ m/s}$$

Yes, safety requirement is met.

[6]



This diagram shows one standard safety railing taken from <https://kusgrp.com/metal-works/product/mild-steel-railing/>

The open drain runs for the length of 47 metres.

For open drains of more than 1.0 m deep, standard safety railings are placed along the side of the drain.

A standard safety railing measures 3000 mm by 1370 mm, with a 50 mm space in between each railing.

Assuming the first railing is placed at the start of the drain, the project manager claims that there is a need to modify the last railing to sufficiently line the open drain safely.

Calculate the length, in mm, of the last railing.

estimate on number of railings needed

$$= \frac{47}{3} = 15\frac{2}{3}$$

$\therefore$ 15 complete railings needed

Length covered by railing including the 50mm space

$$= 15 \times 3 + 15 \times 0.05$$

$$= 45.75 \text{ m}$$

Remaining length not covered by railing

$$= 47 - 45.75$$

$$= 1.25 \text{ m}$$

$$= 1250 \text{ mm}$$

Answer mm [2]