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(i) Without using a calculator, find the exact value of 12^x , given that $3^{2(x+1)} - 4^{1-2x} = 0$. [3]

(ii) Solve the equation $2e^{2x} = 13e^x - 15$. [4]

(i) Find the range of values of x for which $x(10-x) > 24$. [2]

(ii) Find the range of values of c for which $x(10-x) < c^2$. [4]

(i) Sketch, on the same diagram, the graphs of $y = x^{-\frac{1}{2}}$ and $y^2 = 4x$ for $x > 0$. [2]

(ii) Find the coordinates of the point of intersection of the graphs. [3]

(i) Find the exact value of $\sin 165^\circ$. [3]

(ii) Hence, show that $\cot^2 165^\circ$ can be expressed in the form $a + b\sqrt{3}$ where a and b are integers. [4]

Given that the term independent of x in the expansion of $(3+5x^2)\left(1-\frac{1}{2x}\right)^n$ is 38, where n is a constant, find

(a) the value of n , [4]

(b) the coefficient of $\frac{1}{x}$. [3]

The population, P , of a certain species of frogs is given by

$$P = Ae^{-kt},$$

where A and k are constants and t is the time in years from 1 January 2000.

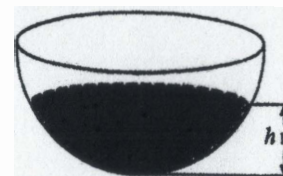
Over a period of 10 years from 1 January 2000 to 1 January 2010, P decreased from 90 000 to 40 000.

Find

(i) the value of A and of k , [3]

(ii) the year in which the population will be reduced by 70% as compared to the year 2000. [2]

7. A hemispherical container is shown in the diagram below.



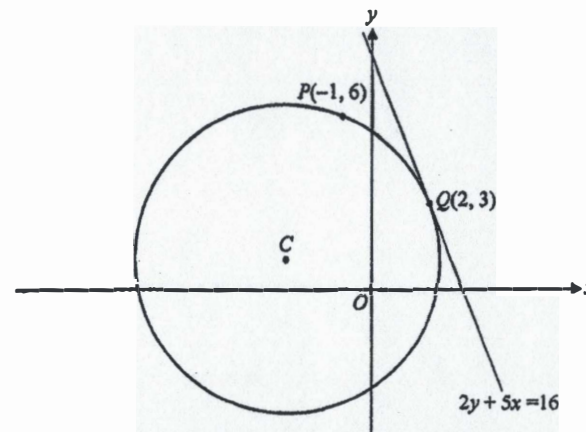
When the depth of the water is h m, the volume, V m³, is given by

$$V = \frac{1}{12}\pi h(3-4h^2), \text{ where } 0 \leq h \leq 0.5.$$

(i) Find the value of h for which $\frac{dV}{dt} = \frac{5\pi}{36} \frac{dh}{dt}$. [3]

(ii) If water is flowing into the bowl at a constant rate of $\frac{\pi}{800}$ m³s⁻¹, find the rate of change of h when $h = 0.25$. [2]

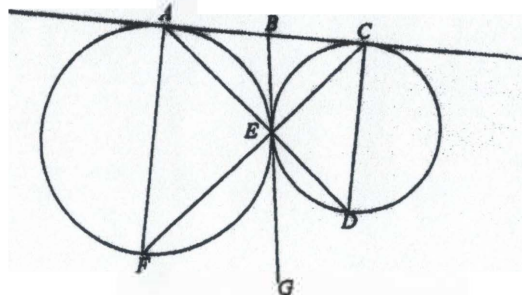
8. A circle, centre C , passes through the point $P(-1, 6)$ and touches the line $2y+5x=16$ at the point $Q(2, 3)$.



(i) Find the equation of the perpendicular bisector of PQ . Hence find the equation of the circle. [6]

(ii) Find the coordinates of R such that $CPQR$ is a parallelogram. [2]

In the diagram, the two circles touch at E . ABC and BEF are common tangents to the two circles. CE is produced to D and F respectively.



Prove that AF is parallel to CD .

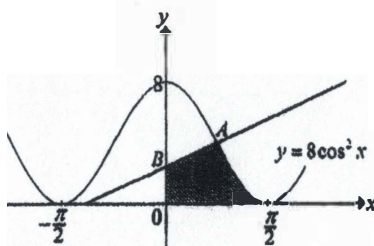
[4]

) Prove that AC is a diameter of a circle which passes through A , E and C .

[4]

) Show that $\frac{d}{dx}(2x + \sin 2x) = 4 \cos^2 x$.

[2]



The diagram shows part of the graph of $y = 8 \cos^2 x$. The normal to the curve at A , where $x = \frac{\pi}{4}$, meets the y -axis at B .

(i) Show that the y -coordinates of B is $\frac{128 - \pi}{32}$.

[4]

(ii) Determine the area of the shaded region bounded by the curve, the line AB , the x -axis and the y -axis.

[5]

11. It is given that $f(x) = x^2 - 8x + 9$ for $2 \leq x \leq 7$.

(i) Find the value of a and of b for which $f(x) = (x - a)^2 + b$.

[2]

(ii) Find the stationary point of the graph $y = |f(x)|$ and determine its nature.

[2]

(iii) Sketch the graph of $y = |f(x)|$.

[3]

(iv) Find the range of values of x for which $|f(x)| > 6$.

[2]

(v) Determine the number of solutions of the equation $|f(x)| = mx + c$ in each of the following cases, when

(a) $m = 1$ and $c = 2$,

[1]

(b) $m = -\frac{1}{2}$ and $c = 4$.

[1]

End of Paper 1

Find a quadratic equation for which the sum of roots is $\frac{1}{2}$ and the sum of the cube of the roots is $\frac{13}{8}$. [5]

(a) Variables x and y are connected by the equation $\log_3 y = a \log_3 x + b$, where a and b are constants. Using experimental values of x and y , a graph was drawn in which $\log_3 y$ was plotted on the vertical axis against $\log_3 x$ on the horizontal axis. The straight line which was obtained passed through the points $(1, 3)$ and $(-1, 5)$.

(i) Find the value of a and of b . [3]

(ii) Show that x and y can be expressed in the form $y = kx^n$, where k and n are constants to be found. [3]

(b) Given that $\log_8 x^3 = \log_{\sqrt{2}} u$, express u in terms of x . [3]

(i) Show that $\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = \frac{1 + \tan x}{1 - \tan x}$. [3]

(ii) Hence, solve for $-3 < x < 2$, the equation $\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = 6 \tan x$. [5]

(a) Find the value of m , where $m > 0$, for which $2x^2 + x + m$ is a factor of $4x^3 + 5x - 3$. [3]

(b) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 3 and the roots of the equation $f(x) = 0$ are -2 , 3 and k . Given that $f(x)$ has a remainder of 42 when divided by $(x+1)$, find

(i) the value of k , [3]

(ii) the remainder when $f(x)$ is divided by x . [2]

(i) Express $\frac{-2x-6}{(x+1)(x^2-3)}$ in partial fractions. [4]

(ii) Differentiate $\ln(x^2 - 3)$. [1]

(iii) Given that $\int_2^3 \frac{-3x-9}{(x+1)(x^2-3)} dx = \frac{9}{2} \ln a$, using the results in parts (i) and (ii), find the value of a . [4]

6. A device is used to simulate the breathing patterns of a certain mammal's lungs. The volume, V litres, of air in the lungs of this mammal, t seconds after the beginning of one breath can be modelled by

$$V = 0.45 - 0.4 \cos(kt), \quad 0 \leq t \leq 4.$$

The time for one breath is 4 seconds.

(i) Explain why this model suggests that the maximum capacity of the lungs is 0.85 litres. [1]

(ii) Show that the value of k is $\frac{\pi}{2}$. [2]

(iii) Find the length of time for which the lungs contain at least 0.5 litres of air. [3]

(iv) Sketch the graph of $V = 0.45 - 0.4 \cos(kt)$, $0 \leq t \leq 4$. [2]

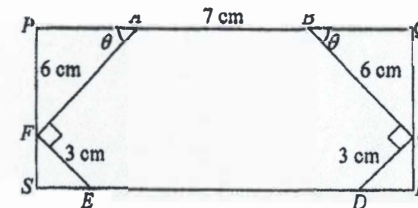
7. A curve has the equation $y = 4x^3 e^{2x}$. It has a stationary point at $\left(p, \frac{q}{e^3}\right)$ where $p < 0$.

(i) Find the exact value of p and of q . [5]

(ii) By considering the sign of $\frac{dy}{dx}$, determine the coordinates and the nature of the other stationary point. [2]

(iii) Find the range of values of x for which $y = 4x^3 e^{2x}$ is a decreasing function. [2]

8. In the diagram, $PQRS$ is a rectangle. $ABCDEF$ is a hexagon with angle $AFE = \text{angle } BCD = 90^\circ$. $AB = 7 \text{ cm}$, $BC = AF = 6 \text{ cm}$, $CD = EF = 3 \text{ cm}$ and angle $QBC = \text{angle } PAF = \theta$, where $0^\circ \leq \theta \leq 90^\circ$.

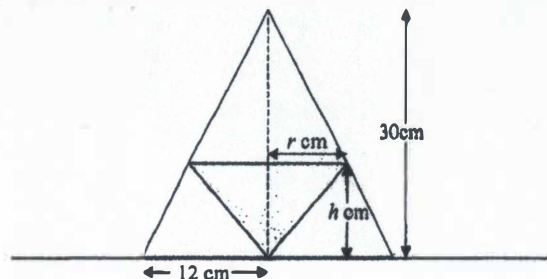


(i) Show that the perimeter, L cm, of $ABCDEF$ is given by $L = 32 + 12 \cos \theta - 6 \sin \theta$. [3]

(ii) Express L in the form $k + R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Find the value of θ , if $L = 35$. [3]

9. The diagram shows the cross-section of a hollow cone of height 30 cm and base radius of 12 cm and an inverted cone of radius r cm and height h cm. Both stand on a horizontal surface with the inverted cone inside the hollow cone. The upper circular edge of the inverted cone is in contact with the hollow cone.



- (i) Express h in terms of r and hence show that the volume, V cm³ of the inverted cone is given by

$$V = \pi \left(10r^3 - \frac{5r^3}{6} \right) \quad [4]$$

Given that r can vary,

- (ii) find, in terms of π , the volume of the largest inverted cone which can stand inside the hollow cone, and show that, in this case, the inverted cone occupies $\frac{4}{27}$ of the volume of the hollow cone. [7]

10. The population P , in millions, of a country was recorded in various years and the results are shown in the table below.

Year	2000	2005	2010	2015
P	12.88	14.61	17.38	21.88

It is known that P and t are related by an equation of the form $P = 10 + Ab^t$, where t is the time measured in years from January 1995 and A and b are constants.

- (i) Using graph paper, draw a straight line graph of $\lg(P - 10)$ against t and use your graph to estimate the value of A and of b . [7]

Use your graph to estimate

- (ii) the population, in millions, in the country in January 1995, [2]
 (iii) the year in which the population exceeds 35 million. [2]

11. The velocity, v ms⁻¹, of a particle, P , moving in a straight line is given by $v = 3t^2 + pt + q$, where t is the time in seconds after the start of motion. At $t = 0$, the displacement of the particle from O is 3 m.

Given also that when $t = 2$, the displacement of the particle from O is 23 m and the acceleration of the particle is -6 ms⁻²,

- (i) find the value of p and of q , [7]
 (ii) explain with clear working whether P will return to its starting point. [5]

End of Paper 2

2016 SCGS Prelim Paper 1

1. (i) $\frac{2}{3}$
(ii) 0.405, 1.61
2. (i) $4 < x < 6$
(ii) $a < -5$ or $c > 5$
3. (ii) $\left(\frac{1}{2}, \sqrt{2}\right)$
4. (i) $\frac{\sqrt{6} - \sqrt{2}}{4}$
(ii) $7 + 4\sqrt{3}$
5. (a) 8
(b) -47
6. (i) 0.0811
(ii) 2014
7. (i) $\frac{1}{3}$
(ii) $\frac{1}{150} \text{ ms}^{-1}$ or 0.00667 ms^{-1}
8. (i) $(x+3)^2 + (y-1)^2 = 29$
(ii) (0, -2)
10. (bii) 4.24 units^2
11. (i) $a = 4$, $b = -7$
(ii) (4, 7), Maximum point
(iv) $3 < x < 5$
(va) 2 solutions
(vb) 3 solutions

2016 SCGS Prelim Paper 2

1. $x^2 - \frac{x}{2} - 1 = 0$
2. (ai) $a = -1$, $b = 4$
(ii) $y = 81x^{-1}$
(b) $u = \sqrt{x}$
3. (ii) $x = -2.82, -2.68, 0.322, 0.464$
4. (a) 3
(bi) $k = \frac{5}{2}$
(bli) 45
5. (i) $\frac{2}{x+1} - \frac{2x}{x^2-3}$
(ii) $\frac{2x}{x^2-3}$
(iii) $\frac{2}{3}$
6. (iii) 1.84 s
7. (i) $p = -\frac{3}{2}$, $q = -\frac{27}{2}$ or -13.5
(ii) point of inflexion
(iii) $x < -\frac{3}{2}$
8. (ii) $L = 32 + 6\sqrt{5} \cos(\theta + 26.6^\circ)$
(iii) 50.5°
9. (ii) $\frac{640\pi}{3} \text{ cm}^3$
10. (ii) 11.8
(iii) 2023
11. (i) $p = -18$, $q = 24$
(ii) $L = 29 + \sqrt{3362} \cos(\theta - 45^\circ)$