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BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2015

SUBJECT : Mathematics

LEVEL : Secondary 4E/5N

PAPER : 4016 / 01

DURATION : 2 hours

SETTER : Mr Bernard Lee

DATE : 1 September 2015

CLASS :	NAME :	REG NO :
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READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use
80

This paper consists of 17 printed pages (including this cover page)

H1

[Turn over

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

- 1 Write the following in order of size, smallest first.

$$\sqrt[3]{-\frac{8}{27}}, -0.6, -0.7, -\frac{19}{30}$$

†

Answer [2]
smallest largest

- 2 (a) Calculate $\frac{0.123^2}{\sqrt[3]{52} - 3.7}$.

Write down the first five digits of your answer.

Answer (a) [1]

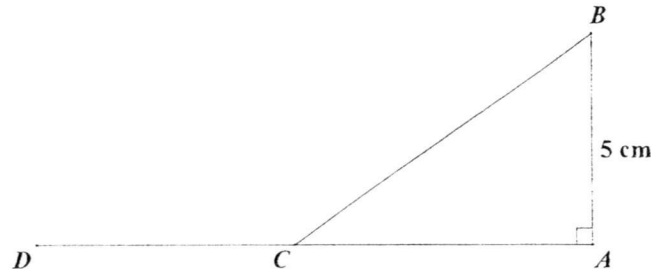
- (b) Write your answer to **part (a)** correct to 2 significant figures.

Answer (b) [1]

- 3 Given that $16^k \times 0.5 = 2^{2015}$, find the value of k .

Answer $k =$ [2]

- 4 In $\triangle ABC$, $AB = 5$ cm and $\angle CAB = 90^\circ$. AC is produced to D .



Given that $\sin \angle BCD = \frac{3}{5}$,

- (a) show that $BC = 8\frac{1}{3}$ cm,

Answer (a)

[1]

- (b) calculate the exact length of AC ,

Answer (b) cm [1]

- (c) calculate $\cos \angle BCD$, giving your answer as a fraction.

Answer (c) [1]

- 5 The length of each edge of a cube is increased by 50%.

Calculate the percentage increase in its volume.

Answer % [2]

6

- (a) The line graph below shows the profits that a company has made over a few years.

Profits (in thousands)



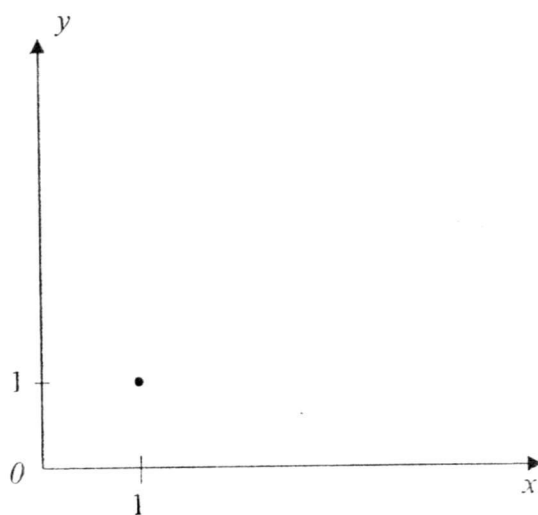
Explain one way in which the graph is misleading.

Answer

.....

..... [2]

- (b) On the axes below, sketch the graph of $y = 3(2^x)$



[1]

7

Loch Ness is a large, deep, freshwater lake in the Scottish Highlands.

- (a) Its deepest point is 230 000 cm. Express 230 000 in standard form.

Answer (a) [1]

- (b) Loch Ness has a volume of 7.5 km^3 . Convert 7.5 km^3 to cm^3 , expressing your answer in standard form.

Answer (b) cm^3 [1]

- (c) Brenda Sherratt was the first person to swim 36.2 km along the length of Loch Ness at an average speed of 1.15 km/h in July 1966. Calculate the number of hours and minutes she took, correct to the nearest minute.

Answer (c) hours minutes [1]

- 8 The volume, $V \text{ cm}^3$, of an object is directly proportional to the cube of its diameter, $d \text{ cm}$. Given that $V = 32.5$ when $d = 5$,

- (a) find a formula connecting V and d ,

Answer (a) [2]

- (b) calculate the diameter of the object when it has a volume of 89 cm^3 ,

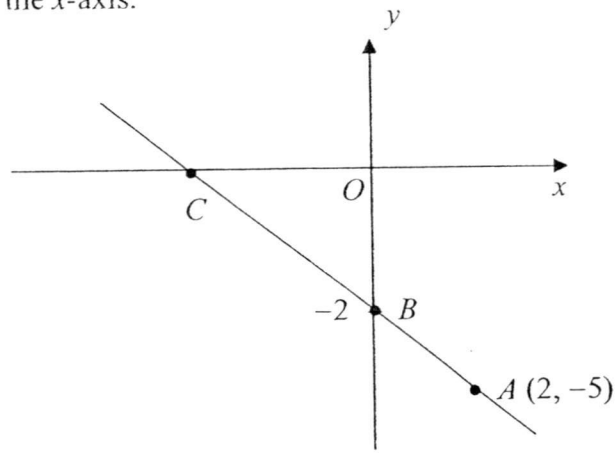
Answer (b) cm [1]

- (c) write down a possible shape of the object, giving a reason for your answer.

Answer (c)
..... [1]

9

The diagram shows a sketch of a straight line passing through the points $A(2, -5)$, B on the y -axis and C on the x -axis.



- (a) Find the equation of the line.

Answer (a) [2]

- (b) Find the coordinates of C .

Answer (b) C (.....,) [1]

- 10 Each term in a sequence is found by multiplying the same constant to the previous term.

$p,$ $12,$ $q,$ $48,$ $r,$...

- (a) Write down 2 possible values of q .

Answer (a) $q =$ or [1]

- (b) Write down the ratio $\frac{p}{r}$.

Answer (b) [1]

- (c) Explain why 386 is not a term in this sequence.

111

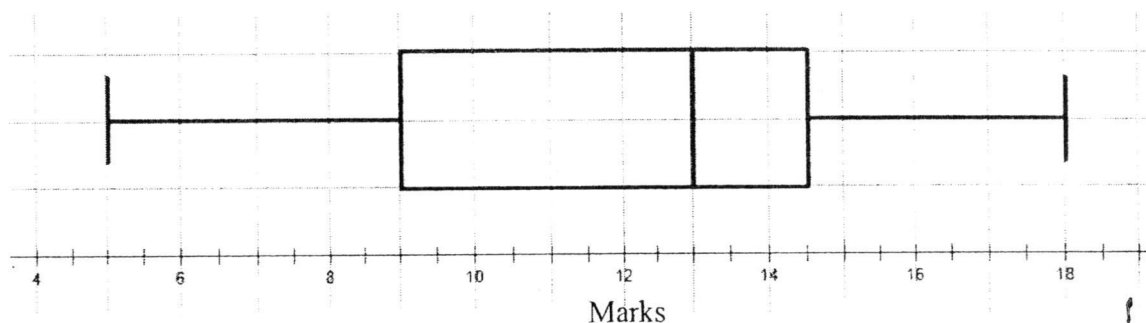
Answer (c)

..... [1]

[Turn over

11

The marks obtained by a class of students in a test are represented by the box-and-whisker plot below.



- (a) Find the interquartile range of the marks.

Answer (a) [1]

- (b) Their marks were arranged in ascending order.
Only 1 student scored 13 marks, and this student was the 15th student in the arranged list.
Calculate the number of students in the class.

Answer (b) [1]

- 12 Alvin wants to change 300 Singapore Dollars (S\$) to Thai Baht (THB).
The exchange rate is S\$1 = 24.95 THB.
For S\$300, the money changer gives Alvin the maximum number of 1000 THB notes that can be bought and returns Alvin the change in S\$, correct to the nearest 10 cents.

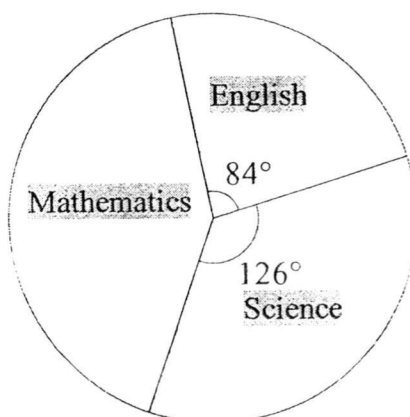
Calculate the amount of THB and S\$ that Alvin will receive.

Answer THB and S\$ [3]

- 13** The cash price of a television set is \$1 260.
The hire-purchase scheme requires the buyer to pay a deposit of 20% of the cash price plus 18 equal monthly instalments of \$59.36.
Calculate the interest rate per annum of the hire-purchase scheme.

Answer % [3]

- 14** Some students were surveyed and asked to write down their favourite subject.
The results were presented in the pie chart below.



- (a)** Calculate the ratio of the number of students who chose Mathematics to the number of students who chose English.

Answer (a) : [2]

- (b)** Calculate the smallest possible number of students who were surveyed.

45

Answer (b) [2]

[Turn over]

For
aminer's
Use

- 15 Ivan invested some money in a savings account for 2 years.
The rate of compound interest was fixed at 10% per annum compounded half-yearly.
At the end of the 2 years there was \$9724.05 in his account.

For
Examine
Use

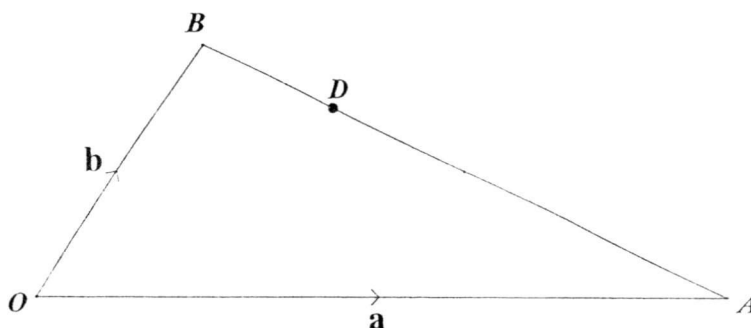
How much did Ivan invest in the account?

Answer \$ [2]

- 16 OAB is a triangle.

D is the point on BA such that $BD = \frac{2}{7} BA$.

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.



- (a) Find $\vec{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer (a) [1]

- (b) E is the point on OA such that $5OE = 2EA$.
Using vectors, determine if ED is parallel to OB .

Answer (b)

[2]

For
minor's
Use

- 17 $\xi = \{\text{positive integers } x: 2x < 22\}$
 $A = \{\text{factors of } 60\}$
 $B = \{\text{prime numbers}\}$

(a) Draw a Venn diagram to illustrate the information.

Answer (a)

[2]

(b) List the elements contained in the set $A \cap B'$.

Answer (b) [1]

18 Simplify

(a) $\frac{1}{x^2 - 1} - \frac{1}{x - 1},$

Answer (a) [3]

(b) $3\sqrt{x^{-2}y^4} \div 12x^5y^{-4},$ leaving your answer in positive index.

46

Answer (b) [2]

[Turn over

For
amateur's
Use

19 (a) Chris travels from Town A to Town B at a constant speed of u km/h.

(i) Convert u km/h into m/s, leaving your answer in terms of u .

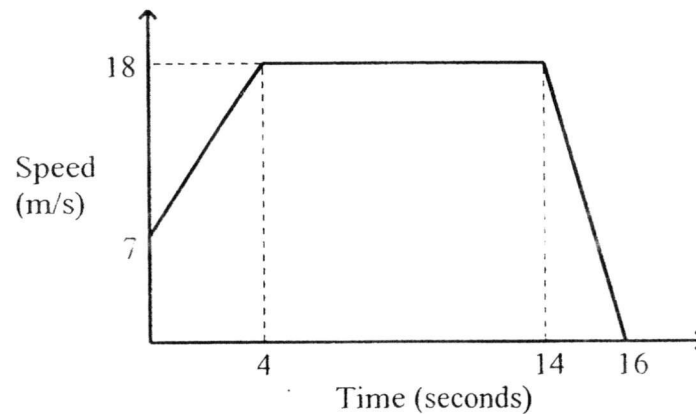
For
Examine
Use

Answer (i)..... m/s [1]

(ii) Steven sets off from Town B for Town A at the same time as Chris at v km/h. Given that the distance between the two towns is 250 km, express, in terms of u and v , the distance from Town A where they will meet.

Answer (ii)..... km [2]

(b) The diagram shows the speed-time graph for a car journey between two points.



Calculate the average speed of the journey.

Answer (b)..... m/s [2]

20

Most stop signs around the world are in the shapes of regular octagons. Some of these stop signs have diameters of 750 mm, i.e. $AB = 750$ mm, as shown in the figure below.



- (a) Calculate the area of the stop sign, leaving your answer in square centimetres.

Answer (a)..... cm^2 [2]

- (b) Calculate the length of one side of the stop sign, leaving your answer in centimetres.

47

Answer (b)..... cm [2]

21

A bag contains 4 balls, numbered 1, 2, 3 and 4.

Two balls are taken from the bag at random, one after the other, **without replacement**.

- (a) Draw a possibility diagram to represent the outcomes.

Answer (a)

†

[2]

- (b) Find, in its simplest form, the probability that
- (i) the product of the two numbers is a square number.

Answer (b)(i)..... [1]

- (ii) one number is odd and another number is even.

Answer (b)(ii)..... [1]

- (iii) the sum of the numbers is more than 5.

Answer (b)(iii)..... [1]

22 (a) (i) Express 594 as a product of its prime factors.

(ii) Find the smallest positive integer k such that $\frac{594}{\sqrt{k}}$ is a perfect cube.

Answer (a)(i) [1]

(ii) $k =$ [1]

(b) Megan is playing with 594 cubes. Megan uses all 594 cubes to make a cuboid. Each of the sides of the cuboid is made up of more than 3 cubes.

Find the number of cubes on each side of the cuboid.

Answer (b)cubes bycubes by.....cubes [1]

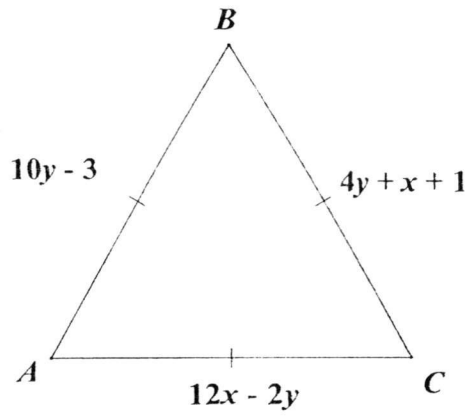
(c) Each cube has a volume of 2 cm^3 . Calculate the maximum number of cubes that Megan can fit into a box of dimensions 8 cm by 10 cm by 10 cm.

Answer (c) [2]

For
aminer's
Use

- 23 The diagram shows an equilateral triangle of lengths $(4y + x + 1)$ cm, $(10y - 3)$ cm and $(12x - 2y)$ cm.

For
Examine
Use



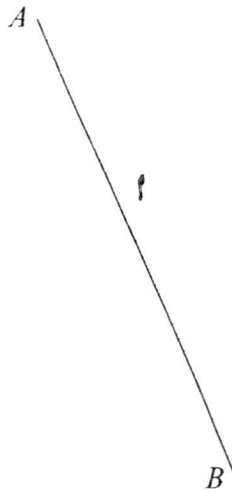
By solving for x and y , calculate the area of the triangle.

Answer cm^2 [5]

- 24 (a) Construct triangle ABC where $BC = 8$ cm and $AC = 10$ cm.
 AB has already been drawn.

[1]

Answer (a), (b), (c) and (d).



- (b) Construct the perpendicular bisector of AB . [1]
- (c) Construct the bisector of angle ABC . [1]
- (d) Mark clearly a possible point which is inside the triangle, equidistant from BC and BA , and is nearer to A than B . [1]
- Label this point P .

For
aminer's
Use**Answer Key**For
Examine
Use

1 (a) $-0.7, \sqrt[3]{-\frac{8}{27}}, -\frac{19}{30}, -0.6$

2 (a) 0.4653 (b) 0.47

3 504

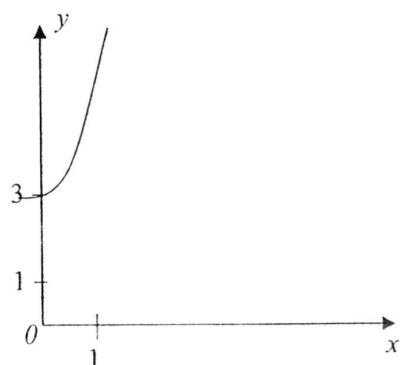
4 (a) Show (b) $6\frac{2}{3}$ (c) $-\frac{4}{5}$

5 237.5%

6 (a) Either one of the misleading feature + effect

Misleading Feature	Effect
Inconsistent scale on horizontal axis	Exaggerates the differences between the years
Not all years are shown	Profits could be very low from 2009 to 2014
Unequal spacing of years	Misrepresents the trend
Vertical axis does not start from 0	Rate of gain in profits appear steeper

6 (b)



7 (a) 2.3×10^5 (b) 7.5×10^{15} (c) 31 hours 29 minutes

8 (a) $V = 0.26d^3$ (b) 7.00 cm

8 (c) Hemisphere. Volume of hemisphere when $d = 5$ is $32.7 \text{ cm}^3 \approx 32.5 \text{ cm}^3$.

9 (a) $y = -\frac{3}{2}x - 2$ (b)(i) $(-\frac{4}{3}, 0)$

10 (a) 24 or -24 (b) $\frac{1}{16}$

10 (c) The next few terms in the sequence are 96, 192, 384, 768, hence 386 is not a term in the sequence

11 (a) 5.5 (b) 29

12 7000 THB and S\$19.40

13 4%

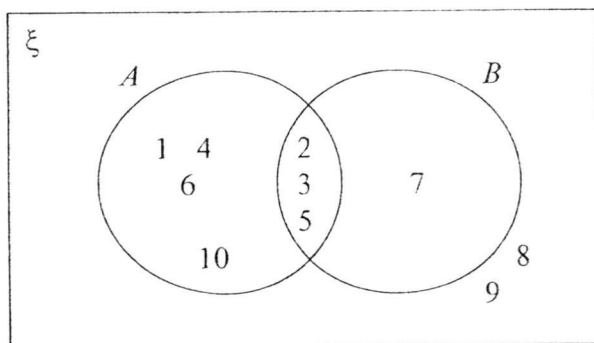
14 (a) 25 : 14 (b) 60

15 \$8000

16 (a) $\frac{5}{7}\mathbf{b} - \frac{5}{7}\mathbf{a}$

(b) Since $\overrightarrow{ED} = \frac{5}{7}\overrightarrow{OB}$, ED is parallel to OB.

17 (a)



17(b) $\{1, 4, 6, 10\}$

18 (a) $\frac{-x}{(x+1)(x-1)}$

(b) $\frac{y^6}{4x^6}$

19 (a)(i) $\frac{5u}{18}$

(a)(ii) $\frac{250u}{u+v}$

(b) 15.5 m/s

20 (a) 3980 cm^2

(b) 28.7 cm

21 (a)

	1	2	3	4
1		(2, 1)	(3, 1)	(4, 1)
2	(1, 2)		(3, 2)	(4, 2)
3	(1, 3)	(2, 3)		(4, 3)
4	(1, 4)	(2, 4)	(3, 4)	

21 (b)(i) $\frac{1}{6}$

(b)(ii) $\frac{2}{3}$

(b)(iii) $\frac{1}{3}$

22 (a)(i) $2 \times 3^3 \times 11$

(a)(ii) 484

(b) 6 by 9 by 11

(c) 294

23 8.77 cm^2

24 Construction



BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2015

SUBJECT : Mathematics

LEVEL : Sec 4 Express

PAPER : 4016 / 02

DURATION : 2 hours 30 minutes

SETTER : Ms Chai YL

DATE : 2 September 2015

CLASS :	NAME :	REG NO :
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Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is **100**.

*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

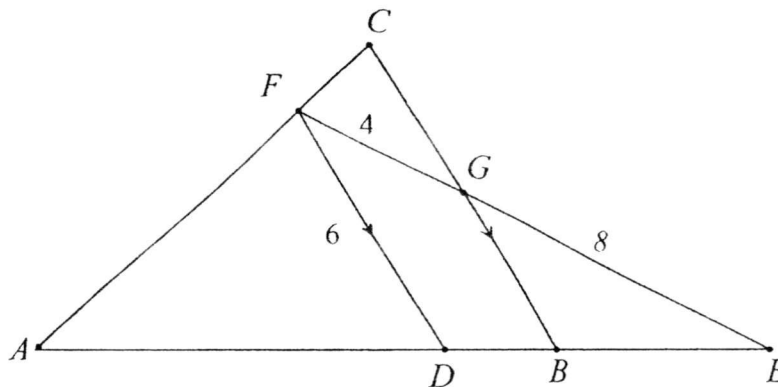
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

- 1 (a) Solve the inequality $3x + 10 \leq \frac{2}{3}(2x + 25)$. [2]
- (b) Given that $R = \frac{3}{t} - \frac{2t}{s}$,
- (i) find R when $t = -5$ and $s = 3$, [1]
- (ii) express s in terms of R and t . [2]
- (c) Simplify $\frac{4p^2 - 36}{5p^2 + 8p - 21}$. [2]
- (d) Solve the equation $\frac{y+2}{4} = \frac{8}{y-2}$. [3]

- 2 In the diagram, BC is parallel to DF , $DF = 6$ cm, $GE = 8$ cm and $FG = 4$ cm.
- (a) Prove that triangles BEG and DEF are similar. [2]
- (b) Hence calculate the length of BG . [2]
- (c) If $AC = 4FC$, find the length of BC . [2]
- (d) If the area of ADF is 20 cm^2 , find the area of $BDFG$. [2]



- 3 (a) Part of the utilities bill for the month of July of a household is shown below.

Water Services by Public Utilities Board			
	Usage	Rate	Amount (\$)
Water	4.7 Cu M	\$ 1.1700	5.50
Waterborne Fee	4.7 Cu M	\$ 0.2803	
Water Conservation Tax		30%	
⋮			
Goods & Services Tax (GST)		7%	

Water Conservation Tax is imposed as a percentage of the total water consumption to reinforce the message that water is precious.

- (i) Find the Water Conservation Tax for this bill. [1]
- (ii) Calculate the total charges for water usage for this household, including GST. [2]
- (iii) The value in part (ii) is 12 % of the entire utilities bill. Find the total charges for the month of July. [2]
- (b) A supermarket has three check-out counters manned by Abby, Brian and Chloe. The following table shows the number of notes of denomination \$2, \$10 and \$50 in their cash registers at the end of a business day.

	Denomination		
	\$2	\$10	\$50
Abby	12	10	25
Brian	20	15	30
Chloe	18	8	24

- (i) Given that $P = \begin{pmatrix} 12 & 10 & 25 \\ 20 & 15 & 30 \\ 18 & 8 & 24 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 \\ 10 \\ 50 \end{pmatrix}$, evaluate PQ and explain what the elements in this matrix product represent. [2]

The three cashiers started the day with \$150, \$100 and \$180 in their cash registers respectively.

- (ii) Write down a (3×1) matrix S to represent these amounts. [1]
- (iii) Write down another matrix T such that a matrix operation involving P , Q , S and T will give the total sales of the supermarket for the day. Hence find this amount. [2]

- 4 A factory producing bottled drinks mixes two different juices, X and Y , by piping in the required amounts using taps of two sizes, small and large, and the mixture is stored in a tank of capacity 2 m^3 .

The small tap dispenses X at the rate of $x \text{ m}^3$ per minute while the large tap dispenses Y at the rate of $y \text{ m}^3$ per minute.

The tank is filled completely in $3\frac{1}{3}$ minutes when the taps are turned on together.

- (a) Show that $y = \frac{3}{5} - x$. [2]
- (b) When used alone, the small tap will take 4 minutes longer than the large tap to fill the tank. Form an equation in x and show that it reduces to $10x^2 - 16x + 3 = 0$. [3]
- (c) Solve the equation $10x^2 - 16x + 3 = 0$, giving your answers correct to 3 significant figures. [3]
- (d) Calculate, to the nearest minute, the time taken by the large tap to fill the tank. [2]
-

- 5 (a) The position vector of point A is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

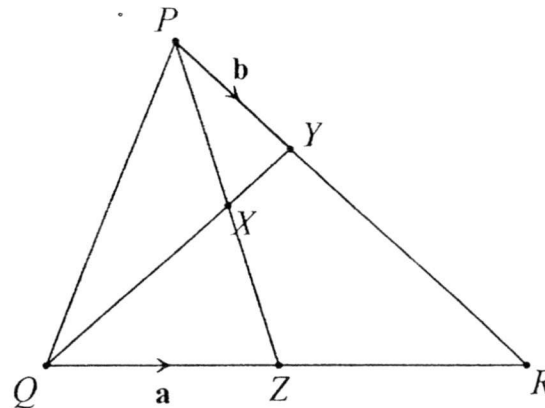
(i) Find $|\overrightarrow{AB}|$. [1]

(ii) Find the coordinates of B . [2]

(iii) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$, and $\overrightarrow{CD} = \begin{pmatrix} k \\ 13.6 \end{pmatrix}$, find the value of k . [2]

- (b) In the diagram, $PR = 3PY$, X is the midpoint of PZ and Z is the midpoint of QR .

$\overrightarrow{QZ} = \mathbf{a}$ and $\overrightarrow{PY} = \mathbf{b}$.



- (i) Express in terms of $\mathbf{a}$ and/or $\mathbf{b}$, as simply as possible

(a) $\overrightarrow{PZ}$ [1]

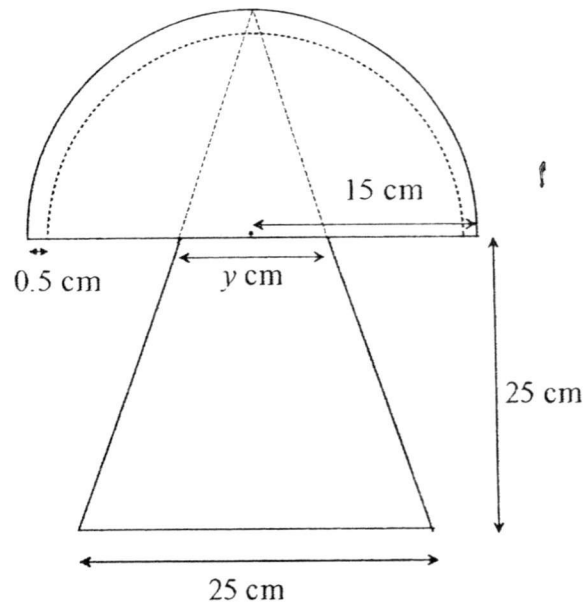
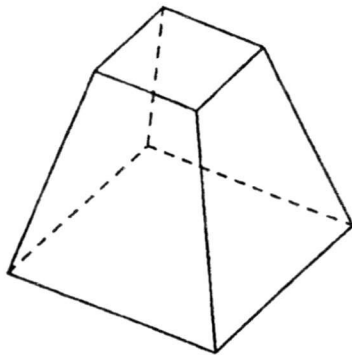
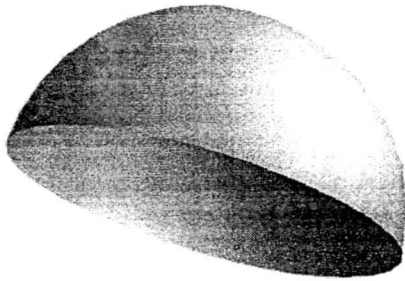
(b) $\overrightarrow{XZ}$ [1]

(c) $\overrightarrow{QX}$ [2]

- (ii) Find the value of $\frac{\text{area of } \triangle PQX}{\text{area of } \triangle PQR}$. [2]
-

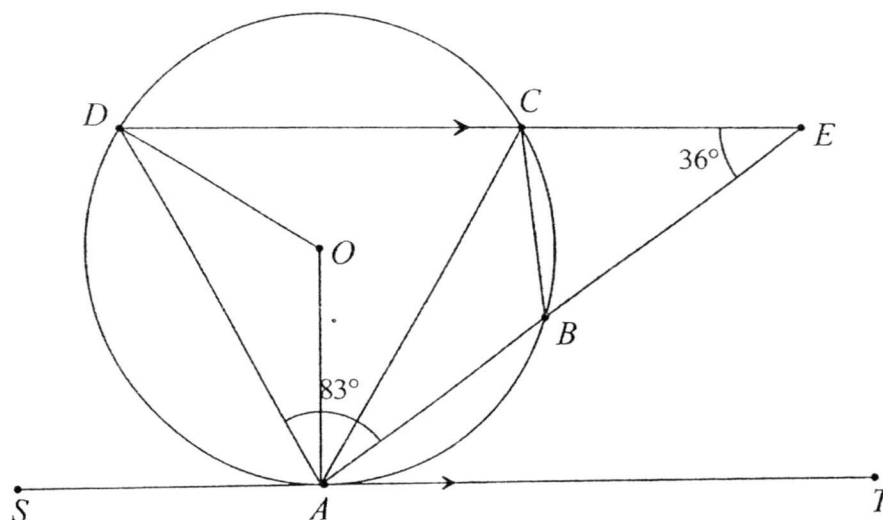
- 6 A mushroom shape table lamp consists of a solid base in the shape of a frustum of height 25 cm, and the lampshade is a hemispherical shell with **external** radius 15 cm. The base of the frustum is a square of side 25 cm.

The parts of the lamp and its cross-section are shown below.



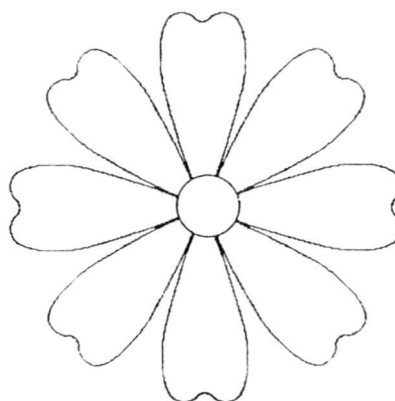
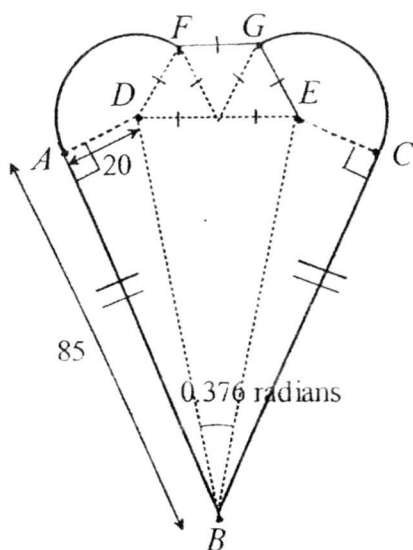
- (a) (i) Show that $y = 9.375$. [1]
 (ii) Calculate the volume of the base. [2]
- (b) The lampshade is made of glass of thickness 0.5 cm. Calculate the volume of glass used to make one such lampshade. [3]
- (c) If a protective layer is applied on the **entire** lampshade, find the total area which needs to be painted. [3]

- 7 (a) In the diagram, A, B, C and D are points on the circumference of the circle centre O . ABE and DCE are straight lines and ST is a tangent to the circle at A . ST is parallel to DE , $\angle CEB = 36^\circ$ and $\angle DAB = 83^\circ$.



- (i) Calculate
- (a) angle OAB , [1]
 - (b) angle ABC , [2]
 - (c) angle AOD . [2]
- (ii) Show that ACD is an isosceles triangle. [1]
- (b) The diagram shows a piece of cardboard used to make a daisy for a carnival float. Eight such pieces are used to make one daisy. Each petal is made up of two sectors, ADF and GEC and six triangles.

The radii of the two sectors are 20 cm, $AB = CB = 85$ cm and $\angle DBE = 0.376$ radians.

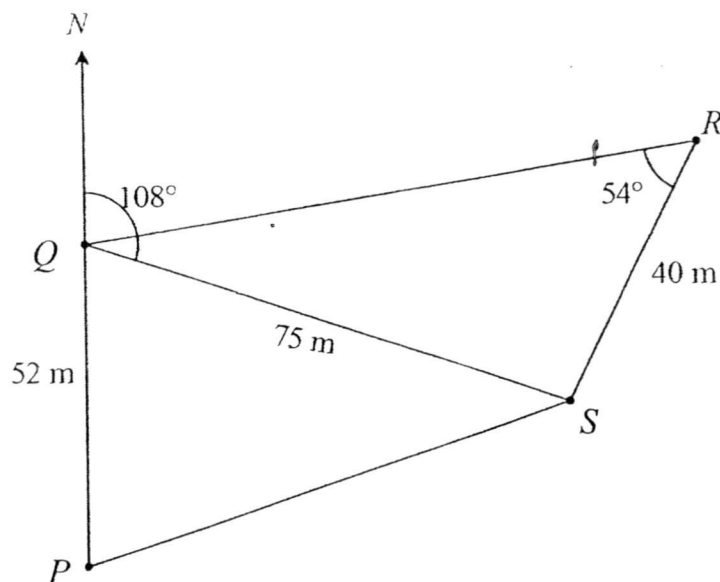


- (i) Calculate $\angle ADB$, giving your answer in radians, correct to 2 decimal places. [2]
- (ii) Find the area of one sector. [4]

- 8 P, Q, R and S are four corners of a plot of land gazetted for development into a neighbourhood park.

$PQ = 52$ m, $QS = 75$ m, $RS = 40$ m and $\angle QRS = 54^\circ$.

P is due south of Q and the bearing of S from Q is 108° .



- (a) Find the distance PS . [2]
- (b) Find the bearing of Q from R . [3]
- (c) Calculate the area of $PQRS$. [2]
- A flag pole is erected at Q and the angle of elevation of the top of this flag pole from P is 13° .
- (d) Calculate the height of the flag pole. [2]

- 9 (a) The lifespan, measured to the nearest hour, of a brand of batteries, *Power Max*, are given in the table below.

Number of hours	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
Number of batteries	5	10	50	20	3

- (i) Calculate the

(a) mean, [1]

(b) standard deviation. [2]

- (ii) Another brand, *Durable*, has a mean lifespan of p hours and a standard deviation of q hours. If p is larger than your answer to (i) (a) and q is smaller than your answer to (i) (b), decide which brand of batteries you prefer and explain your choice. [2]

- (b) On any day, Alice may go to work by bus, MRT or taxi.

The probability that she takes the bus is $\frac{1}{5}$.

The probability that she takes the MRT is $\frac{3}{4}$.

- (i) Find the probability that she goes to work by taxi. [1]

If she takes the bus, the probability that she will be late for work is $\frac{1}{8}$.

If she takes the MRT, the probability that she will be late is $\frac{1}{15}$.

She will not be late if she takes a taxi. Find the probability that

- (ii) Alice will be late for work on any given day, [2]

- (iii) Alice will not be late for two of three consecutive days. [2]
-

10 *Answer the whole of this question on a single sheet of graph paper.*

A group of research scientists brought 320 pelicans to an island. The number of pelicans, y , was counted at intervals and recorded in the table below.

x (years)	0	0.5	1.5	2	2.5	3.5	4.5	5.5	6	7
y	320	266	264	n	351	469	556	554	500	250

- (a) Using a scale of 2 cm to represent 1 year on the x -axis and 2 cm to represent 50 animals on the y -axis for $100 \leq y \leq 600$, plot the values given and join them with a smooth curve. [3]
- (b) Use your curve to estimate
- (i) the value of n , [1]
- (ii) the maximum number of pelicans on the island within these 7 years. [1]
- (c) Give a possible reason why the number of pelicans decreased at the beginning. [1]
- (d) The scientists also brought 100 penguins to the same island, and noticed that the number of penguins increased at a steady rate of 90 per year.
- (i) Write down an equation connecting the number of penguins, y , and the number of years, x . [1]
- (ii) On the same axes, draw the graph of the equation found in (d) (i). [1]
- (iii) Find the value of x when the number of pelicans is equal to the number of penguins. [1]
- (e) Estimate the value of x at the instant of time when the rate of increase of number of pelicans is equal to the rate of increase of the number of penguins, showing clearly how you obtain your answer. [2]

~ ~ End of Paper ~ ~

4E/5N Prelim 2015 Math Paper 2 Ans Key

- 1 (a) $x \leq 4$ (b) (i) $2\frac{11}{15}$ (ii) $s = \frac{2t^2}{3-Rt}$ (c) $\frac{4(p-3)}{5p-7}$
 (d) $y = 6$ or $y = -6$
- 2 (a) $\angle BEG = \angle DEF$ (common angle)
 $\angle GBE = \angle FDE$ (corresponding angles, $BC \parallel DF$)
 Hence BEG and DEF are similar (AA)
 (b) $BG = 4$ cm (c) $BC = 8$ cm (d) $11\frac{1}{9}$ cm²
- 3 (a) (i) \$1.65 (ii) \$9.06 (2 d.p.) (iii) \$75.50 (2 d.p.)
 (b) (i) $\begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix}$ (ii) $S = \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix}$ (iii) $T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, $T(PQ - S) = \begin{pmatrix} 3950 \end{pmatrix}$
 The elements represent the total amount of money in each of the cash registers at the end of the day.
- 4 (a) - (b) - (c) 1.38 or 0.217 (3 s.f.)
 (d) $x = 1.38$ is rejected
 Time taken by large tap = 5 minutes (nearest minute)
- 5 (a) (i) 5 units (ii) B is $(-1, 9)$ (iii) $n = -3.4$, $k = -10.2$
 (b) (i) (a) $\overline{PZ} = 3\mathbf{b} - \mathbf{a}$ (b) $\overline{XZ} = \frac{1}{2}(3\mathbf{b} - \mathbf{a})$ (c) $\overline{QX} = \frac{1}{2}(3\mathbf{a} - 3\mathbf{b})$
 (ii) $\frac{\text{area of } \triangle PQX}{\text{area of } \triangle PQR} = \frac{1}{4}$
- 6 (a) (i) - (ii) 7890 cm³ (3 s.f.) (b) 684 cm³ (3 s.f.) (c) 2780 cm² (3 s.f.)
- 7 (a) (i) (a) 54° (b) 119° (c) 122° (ii) -
 (b) (i) 1.34 radians (2 d.p.) (ii) 503 cm² (3 s.f.)
- 8 (a) 76.9 m (3 s.f.) (b) 262.4° (c) 3330 m² (d) 12.0 m (3 s.f.)
- 9 (a) (i) (a) 47.3 hours (3 s.f.) (b) 4.18 hours
 (ii) *Durable* because the average lifespan is longer since its mean is larger, and lifespan is also more consistent because the standard deviation is smaller.
 (b) (i) $\frac{1}{20}$ (ii) $\frac{3}{40}$ (iii) $\frac{12321}{64000}$

- 10 (b) (i) 300 ± 10 (ii) 570 ± 10 (c) Pelicans were getting used to a new environment
(d) (i) $y = 100 + 90x$ (iii) $x = 5.2 \pm 0.1$
(e) Either 2 ± 0.1 or 4 ± 0.2

4E/5N Prelim 2015 Math Paper 2

1 (a) $3x + 10 \leq \frac{2}{3}(2x + 25)$

$$9x + 30 \leq 4x + 50 \quad [\text{M1}]$$

$$5x \leq 20$$

$$x \leq 4 \quad [\text{A1}]$$

(b) Given that $R = \frac{3}{t} - \frac{2t}{s}$,

(i) find R when $t = -5$ and $s = 3$,

$$R = \frac{3}{-5} - \frac{2(-5)}{3}$$

$$= 2\frac{11}{15} \quad [\text{B1}]$$

(ii) express s in terms of R and t .

$$Rts = 3s - 2t^2$$

$$3s - Rts = 2t^2 \quad [\text{M1}]$$

$$s(3 - Rt) = 2t^2$$

$$s = \frac{2t^2}{3 - Rt} \quad [\text{A1}]$$

(c) $\frac{4p^2 - 36}{5p^2 + 8p - 21}$

$$= \frac{4(p-3)(p+3)}{(5p-7)(p+3)} \quad [\text{M1}]$$

$$= \frac{4(p-3)}{5p-7} \quad [\text{A1}]$$

(d) $\frac{y+2}{4} = \frac{8}{y-2}$

$$y^2 - 4 = 32 \quad [\text{M1}]$$

$$y^2 = 36 \quad [\text{M1}]$$

$$y = 6 \quad \text{or} \quad y = -6 \quad [\text{A1 for both correct answers}]$$

2 (a) $\angle BEG = \angle DEF$ (common angle)

$\angle GBE = \angle FDE$ (corresponding angles, $BC \parallel DF$) [M1]

Hence BEG and DEF are similar (AA) [A1]

58

(b) $\frac{BG}{6} = \frac{8}{12} \quad [\text{M1}]$

$$BG = 4 \text{ cm} \quad [\text{A1}]$$

$$(c) \quad \frac{BC}{6} = \frac{4}{3} \quad [M1]$$

$$BC = 8 \text{ cm} \quad [A1]$$

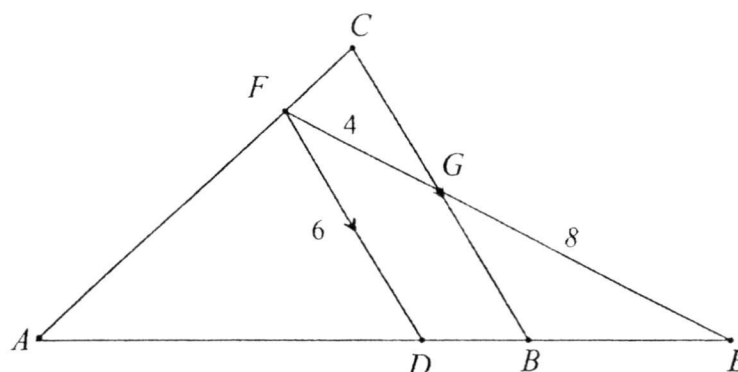
(d) Area of ADF = area of DEF

$$\text{Area of } BEG = \frac{4}{9} (\text{area of } DEF) \quad [M1]$$

$$\text{Area of } BDFG = \frac{5}{9} \times 20$$

$$= 11\frac{1}{9} \text{ cm}^2$$

[A1]



3 (a) (i) Find the Water Conservation Tax for this bill.

$$\text{Water Conservation Tax} = \frac{30}{100} \times \$5.50$$

$$= \$1.65 \quad [B1]$$

(ii) Calculate the total charges for water usage for this household, including GST.

$$\text{Total} = \$ (5.50 + 4.7 \times 0.2803 + 1.65) \times 1.07 \quad [M1]$$

$$= \$ 9.06 \text{ (2 d.p.)} \quad [A1]$$

(iii) The value in part (iii) is 12% of the entire utilities bill. Find the total charges for the month of July.

$$\text{Total charges for July} = \$ 9.0601 \times \frac{100}{12} \quad [M1]$$

$$= \$75.50 \text{ (2 d.p.)} \quad [A1]$$

$$(b) \quad (i) \quad PQ = \begin{pmatrix} 12 & 10 & 25 \\ 20 & 15 & 30 \\ 18 & 8 & 24 \end{pmatrix} \begin{pmatrix} 2 \\ 10 \\ 50 \end{pmatrix}$$

$$= \begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix} \quad [B1]$$

The elements represent the total amount of money in each of the cash registers at the end of the day. [B1]

$$(ii) S = \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix}$$

$$(iii) T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \quad [M1]$$

$$\begin{aligned} T(PQ - S) &= \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \left[\begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix} - \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix} \right] \\ &= (3950) \quad [A1] \end{aligned}$$

4 (a) Express y in terms of x . [2]

$$x + y = \frac{2}{3\frac{1}{3}} \quad [M1]$$

$$= \frac{3}{5}$$

$$\text{Hence } y = \frac{3}{5} - x \quad [A1]$$

$$(b) \frac{2}{x} = \frac{2}{y} + 4 \quad [M1]$$

$$2y = 2x + 4xy$$

$$2\left(\frac{3}{5} - x\right) = 2x + 4x\left(\frac{3}{5} - x\right)$$

$$6 - 10x = 10x + 12x - 20x^2 \quad [M1]$$

$$20x^2 - 32x + 6 = 0 \quad [A1]$$

$$10x^2 - 16x + 3 = 0$$

$$(c) x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(10)(3)}}{2(10)} \quad [M1]$$

$$= \frac{16 \pm \sqrt{136}}{20}$$

$$= 1.3830 \text{ or } 0.21690$$

$$= 1.38 \text{ or } 0.217 \text{ (3 s.f.)} \quad [A1, A1]$$

(d) $x = 1.38$ is rejected

$$\text{Time taken by large tap} = \frac{2}{0.6 - 0.21690} \quad [M1] \quad 5^9$$

$$= 5.22$$

$$= 5 \text{ minutes (nearest minute)} \quad [A1]$$

5 (a) (i) $|\overline{AB}| = \sqrt{(-3)^2 + 4^2}$

$= 5 \text{ units}$ [B1]

(ii) $\overline{OB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ [M1]

$B \text{ is } (-1, 9)$ [B1]

$\overline{CD} = n\overline{BA}$

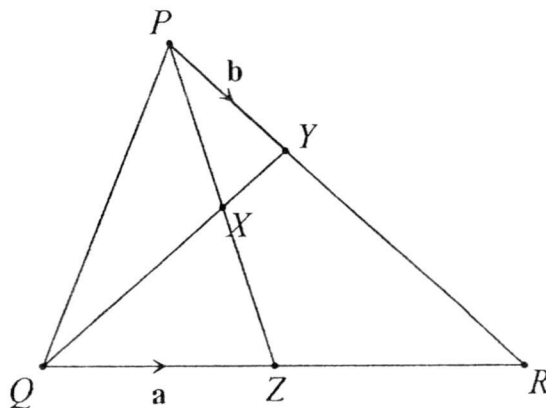
(iii) $\begin{pmatrix} k \\ 13.6 \end{pmatrix} = n \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ [M1]

$n = -3.4$

$k = -10.2$ [A1]

(b) In the diagram, $PR = 3PY$, X is the midpoint of PZ and Z is the midpoint of QR .

$\overline{QZ} = \mathbf{a}$ and $\overline{PY} = \mathbf{b}$.



(i) Express in terms of $\mathbf{a}$ and/or $\mathbf{b}$, as simply as possible

(a) $\overline{PZ} = 3\mathbf{b} - \mathbf{a}$ [B1]

(b) $\overline{XZ} = \frac{1}{2} (3\mathbf{b} - \mathbf{a})$ [B1]

(c) $\overline{QX} = \mathbf{a} - \frac{1}{2} (3\mathbf{b} - \mathbf{a})$ [M1]

$= \frac{1}{2} (3\mathbf{a} - 3\mathbf{b})$ [B1]

(ii) Find the value of $\frac{\text{area of } \triangle PQX}{\text{area of } \triangle PQR}$. [2]

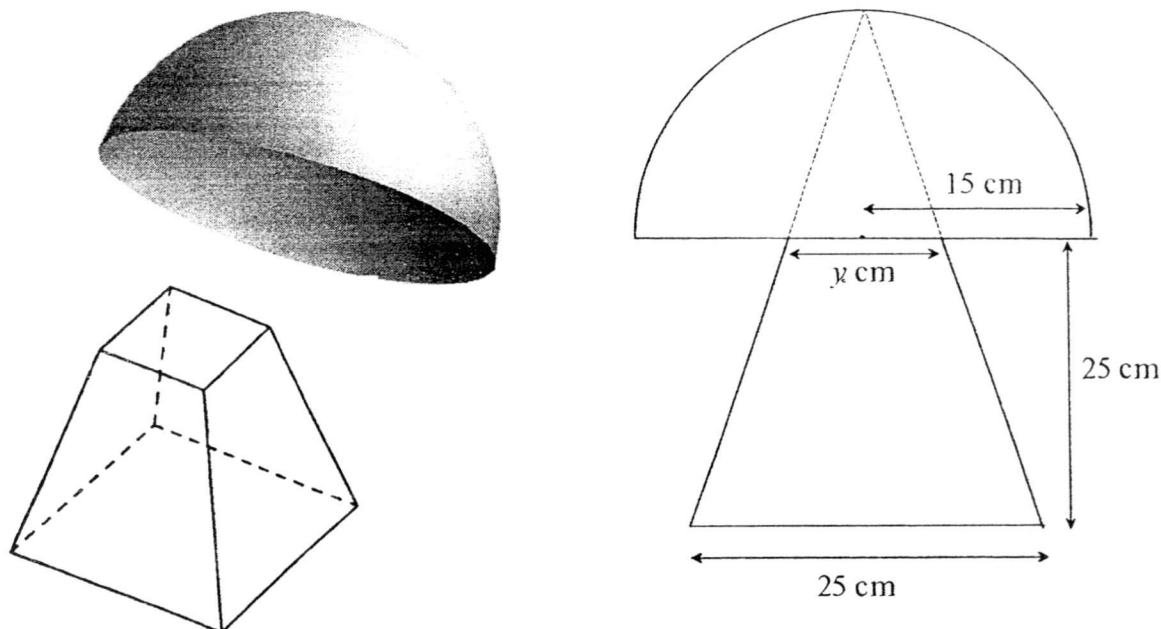
$\triangle PQX = \frac{1}{2} (\triangle PQZ)$

$= \frac{1}{2} \left(\frac{1}{2} \triangle PQR \right)$ [M1]

$\frac{\text{area of } \triangle PQX}{\text{area of } \triangle PQR} = \frac{1}{4}$ [A1]

- 6 A mushroom shape table lamp consists of a solid base in the shape of a frustum of height 25 cm, and the lampshade is a hemispherical shell with **external** radius 15 cm. The base of the frustum is a square of side 25 cm.

The parts of the lamp and its cross-section are shown below.



- (a) (i) Show that $y = 9.375$. [1]

$$\frac{y}{25} = \frac{15}{40} \quad [\text{M1}]$$

$$y = 9.375$$

- (ii) Calculate the volume of the base. [2]

$$\text{Volume of base} = \frac{1}{3}(25)^2(25+15) - \frac{1}{3}(9.375)^2(15) \quad [\text{M1}]$$

$$= 7893.88$$

$$= 7890 \text{ cm}^3 \text{ (3 s.f)} \quad [\text{A1}]$$

- (b) The lampshade is made of glass of thickness 0.5 cm. Calculate the volume of glass used to make one such lampshade. [3]

$$\text{Volume of glass} = \frac{2}{3}\pi(15)^3 - \frac{2}{3}\pi(15-0.5)^3 \quad [\text{M1}, \text{M1}]$$

$$= 683.558$$

$$= 684 \text{ cm}^3 \text{ (3 s.f)}$$

- (c) If a protective layer is applied on the **entire** lampshade, find the total area which needs to be painted. [3]

$$\text{Total area} = 2\pi(15)^2 + 2\pi(14.5)^2 + \pi(15) - \pi(14.5)^2 \quad [\text{M1}, \text{M1}] \quad 60$$

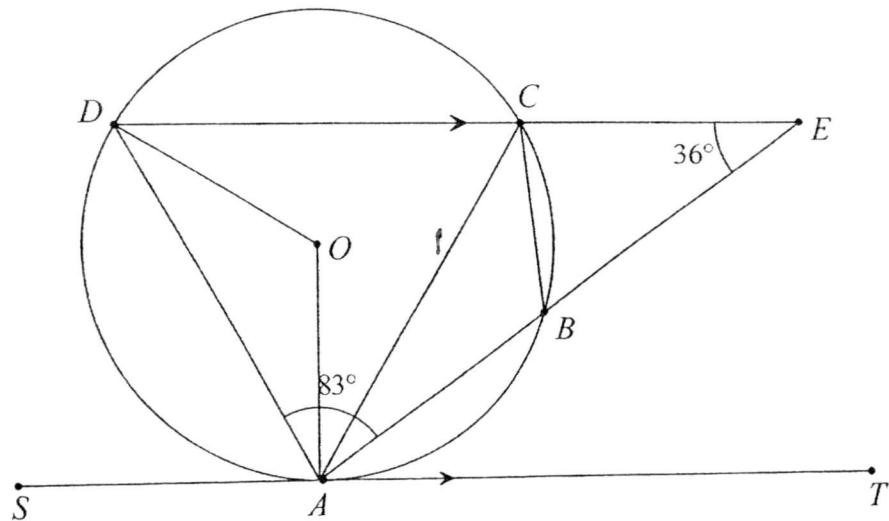
$$= 2781.09$$

$$= 2780 \text{ cm}^2 \text{ (3 s.f)} \quad [\text{A1}]$$

- 7 (a) In the diagram, A, B, C and D are points on the circumference of the circle centre O .

ABE and DCE are straight lines and ST is a tangent to the circle at A .

ST is parallel to DE , $\angle CEB = 36^\circ$ and $\angle DAB = 83^\circ$.



- (i) Calculate

- (a) angle OAB , [1]

$$\angle EAT = 36^\circ \text{ (alt angles, } DE \parallel ST\text{)}$$

$$\angle OAB = 90^\circ - 36^\circ \text{ (tangent perpendicular to radius)}$$

$$= 54^\circ \quad [A1]$$

- (b) angle ABC , [1]

$$\angle ADE = 180^\circ - 36^\circ - 83^\circ \text{ (angle sum of } \Delta\text{)} \quad [M1]$$

$$= 61^\circ$$

$$\angle ABC = 180^\circ - 61^\circ \text{ (angles in opposite segments)}$$

$$= 119^\circ \quad [A1]$$

- (c) angle AOD . [2]

$$\angle OAD = 83^\circ - 54^\circ$$

$$= 29^\circ$$

$$\angle AOD = 180^\circ - 2(29^\circ) \quad [M1]$$

$$= 122^\circ \quad [A1]$$

- (ii) Show that ACD is an isosceles triangle. [1]

$$\angle ACD = \frac{1}{2}(122^\circ) \text{ (angle at centre = twice angle at circumference)}$$

$$= 61^\circ \quad [B1]$$

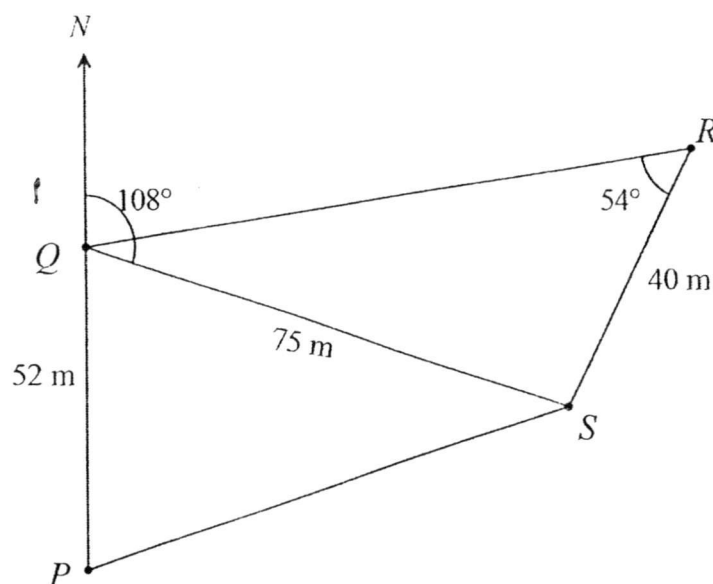
Hence ACD is isosceles.

- (b) The diagram shows a piece of cardboard used to make a daisy for a carnival float. Eight such pieces are used to make one daisy. Each petal is made up of two sectors, ADF and

- 8 P, Q, R and S are four corners of a plot of land gazetted for development into a neighbourhood park.

$PQ = 52$ m, $QS = 75$ m, $RS = 40$ m and $\angle QRS = 54^\circ$.

P is due south of Q and the bearing of S from Q is 108° .



- (a) Find the distance PS . [2]

$$PS^2 = 52^2 + 75^2 - 2(52)(75)\cos 72^\circ \quad [\text{M1}]$$

$$PS = 76.932$$

$$= 76.9 \text{ m (3 s.f.)} \quad [\text{A1}]$$

- (b) Find the bearing of Q from R . [3]

$$\frac{\sin \angle RQS}{40} = \frac{\sin 54^\circ}{75} \quad [\text{M1}]$$

$$\angle RQS = 25.561^\circ$$

$$\text{Bearing of } Q \text{ from } R = 180^\circ + 108^\circ - 25.561^\circ \quad [\text{M1}]$$

$$= 262.4^\circ \quad [\text{A1}]$$

- (c) Calculate the area of this plot of land. [2]

$$\text{Area of land} = \frac{1}{2}(52)(75)\sin 72^\circ + \frac{1}{2}(40)(75)\sin(180^\circ - 54^\circ - 25.561^\circ) \quad [\text{M1}]$$

$$= 3329.7$$

$$= 3330 \text{ m}^2 \text{ (3 s.f.)} \quad [\text{A1}]$$

A flag pole is erected at Q and the angle of elevation of the top of flag pole from P is 13° .

- (d) Calculate the height of the rock wall. [2]

$$\tan 13^\circ = \frac{h}{52} \quad [\text{M1}]$$

$$\text{Height of rock wall} = 52 \times \tan 13^\circ$$

$$= 12.0 \text{ m (3 s.f.)}$$

- 9 (a) (i) Calculate, for these batteries, the

$$\begin{aligned} \text{(a) Mean} &= \frac{4166}{88} \\ &= 47.3 \text{ hours (3 s.f)} \quad [\text{B1}] \end{aligned}$$

$$\begin{aligned} \text{(b) S. D.} &= \sqrt{\frac{198762}{88} - \left(\frac{4166}{88}\right)^2} \quad [\text{M1}] \\ &= 4.18 \text{ hours} \quad [\text{A1}] \end{aligned}$$

- (ii) Another brand, *Durable*, has a mean lifespan of p hours and a standard deviation of q hours. If p is larger than your answer to (i) (a) and q is smaller than your answer to (i) (b), decide which brand of batteries you prefer and explain your choice. [2]

Durable because the average lifespan is longer since its mean is larger, and lifespan is also more consistent because the standard deviation is smaller. [B1 for choice, B1 for explanation]

- (b) On any day, Alice may go to work by bus or MRT with probabilities of $\frac{1}{5}$ and $\frac{3}{4}$ respectively. She may also take a taxi to work on some days.

- (i) Find the probability that she goes to work by taxi. [1]

$$\begin{aligned} P(\text{take taxi}) &= 1 - \frac{1}{5} - \frac{3}{4} \\ &= \frac{1}{20} \end{aligned}$$

If she takes the bus, the probability that she will be late for work is $\frac{1}{8}$.

If she takes the MRT, the probability that she will be late is $\frac{1}{15}$.

She will not be late if she takes a taxi. Find the probability that

- (ii) Alice will be late for work on any given day. [2]

$$\begin{aligned} P(\text{late on any day}) &= \frac{1}{5} \times \frac{1}{8} + \frac{3}{4} \times \frac{1}{15} \\ &= \frac{3}{40} \end{aligned}$$

- (iii) Alice will be punctual for two of three consecutive days. [2]

$$\begin{aligned} P(\text{punctual for 2 out of 3 days}) &= \left(\frac{37}{40} \times \frac{37}{40} \times \frac{3}{40} \right) \times 3 \quad [\text{M1}] \\ &= \frac{12321}{64000} \quad 62 \quad [\text{A1}] \end{aligned}$$

10 *Answer the whole of this question on a single sheet of graph paper.*

A group of research scientists brought 320 pelicans to an island. The number of pelicans, y , was counted at intervals and recorded in the table below.

x (years)	0	0.5	1.5	2	2.5	3.5	4.5	5.5	6	7
y	320	266	264	n	351	469	556	554	500	250

- (a) Using a scale of 2 cm to represent 1 year on the x -axis and 2 cm to represent 50 animals on the y -axis for $100 \leq y \leq 600$, plot the values given and join them with a smooth curve. [3]
- (b) Use your curve to estimate
- (i) the value of n , [1]
 - (ii) the maximum number of pelicans on the island within these 7 years. [1]
- (c) Explain briefly why the number of pelicans decreased at the beginning. [1]
- (d) The scientists also brought 100 penguins to the same island, and noticed that the number of penguins increased at a steady rate of 90 per year.
- (i) Write down an equation connecting the number of penguins, y , and the number of years, x . [1]
 - (ii) On the same axes, draw the graph of the equation found in (d) (i). [1]
 - (iii) Find the value of x when the number of pelicans is equal to the number of penguins. [1]
- (e) Estimate when the rate of increase of number of pelicans is equal to the rate of increase of the number of penguins. [2]

~~ End of Paper ~~